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VICTOR SCHUMANN

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By THEODORE LYMAN

Victor Schumann was born near Leipzig in the year 1841. He received his early education at Leipzig and later, from 1860 to 1864, he was at the Gewerbeschule at Chemnitz. It must have been during this period that he acquired that extraordinary mechanical technique which characterized all his scientific work. For a time he was employed as a designer by Hartmann & Zimmermann; later he was engaged in the manufacture of machinery for the book industry; finally, he became a partner with Mr. A. Hogenforst in the machine business in which he remained actively engaged until 1892 and by means of which he was able to accumulate the funds which he spent in scientific work.

He was more than forty years of age before he was able to turn from his business to scientific pursuits. Even then, his investigations were conducted in the evening or at such odd times as he could spare from his regular profession. Photography first attracted him; one of his earliest papers, published in 1885, is on the sensitization of photographic plates. Almost immediately after this, however, he took up the pursuit of spectrum analysis, to which he devoted himself for the remainder of his active life. His first paper on the spectrum of hydrogen and upon the effect of the presence of impurities on the spectrum of mercury appeared in

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1886.   He must have been inspired by the idea of penetrating into the region of the extreme ultra-violet very early in his scientific studies, for it was but four years later that the article which marks the beginning of his attack on the region of the most refrangible rays appeared.   Guided by the work of Stokes, Soret, and Miller, he began by instituting a very careful comparison of the relative advantages of fluorite and quartz, and, becoming convinced of the superiority of fluorite as a transparent medium for rays of the shortest wave-length, he employed this substance for his prisms and lenses.   Thus equipped, he followed the spark spectra of more than twenty metals to the region λ 1820.   He next set himself the problem of determining the factors which caused the common limit in the spectrum of so many substances.   His knowledge of photographic phenomena led him to recognize the part played by the absorption of gelatine, while his familiarity with the work of Cornu drew his attention to the absorption of the air.   He put these ideas to the test by the construction of a vacuum spectroscope and by the use of special photographic plates whose emulsion was nearly free from gelatine.   His efforts were almost immediately crowned with success, for a very considerable extension of the spectrum followed the use of the new apparatus.   Brief accounts of this work appeared between 1890 and 1893, while a more detailed description of these researches was published in the *Proceedings* of the Vienna Academy in the latter year.   It was during this period that Schumann gave up his business interests to devote himself entirely to his spectroscope.   During the next ten years he went steadily forward, accurately and surely adding to his methods one improvement after another as the results of experiments showed the way, until he finally arrived at the limit of the spectrum set by the absorption of fluorite near λ 1200.

But as early as 1897 his health began to give way.   Never of a robust constitution, he had submitted to considerable privations in early life in order to obtain the funds for the purchase of books. He undoubtedly still further undermined his constitution by the arduous labors entailed in the construction of his apparatus.   In 1903 he was forced to give up nearly all experimental work.   He died on September 1, 1913.

Many of Schumann's results are to be found summed up in the *Smithsonian Contributions to Knowledge*, No. 1413. His first considerable contribution to science was the investigation of the absorption of the air. The existence of the region which bears his name having been once established, he set himself to study the absorption of a number of gases and demonstrated that it was to oxygen that the high absorbing power of the air was due. He next proved that hydrogen possessed great transparency in the most refrangible region and made use of this fact to improve the action of his spectroscope by flushing the interior with this gas. On turning his attention to emission spectra, he obtained valuable information on the radiations from oxygen, carbon monoxide, carbon dioxide, and nitrogen; but it was in the study of the spectrum of hydrogen that his finest results were obtained. He showed that this gas possessed a multitude of lines extending from near λ 1650 to the limit set by fluorite, and by great technical skill and keen experimental insight he succeeded in producing spectrograms of hydrogen which probably will never be surpassed for definition and finish.

Labor spent in the extension of human knowledge is never wasted, no matter how remote from active human interest the field of such labor may appear. The work of Schumann is a brilliant example of the truth of this statement. For the region into which he penetrated reveals day by day to those who explore it greater and greater possibilities for results of fundamental scientific importance. The biologist may watch wonderful changes in living organisms if he will illuminate the field of his microscope with the extreme ultra-violet rays from a hydrogen tube; the student of spectral series may find the key to his fascinating problem on the more refrangible side of λ 1500, and the mathematical physicist who seeks to verify the quantum theory by photo-electric action will find an important test for his hypothesis in the Schumann region.

It has been said that genius consists in an illimitable power of taking pains. Schumann's genius belonged to this character, but the observer who, having marveled at the intricate construction of his spectroscope, obtains the impression that its mechanical perfection represents the highest mental attainment of its maker

entirely misses the truth. Schumann took up the pursuit of science at a time of life when initiative and perseverance in most men are no longer active qualities. The extent of his studies was cut short by the failure of his health. It was never given to him to explore thoroughly the promised land which he discovered. But Schumann possessed the mind of the true investigator; his inductive reasoning was without a flaw. What he did, he did excellently. The final results of his labors are established so firmly that they will never be shaken.

JEFFERSON LABORATORY
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TERTIARY STANDARDS WITH THE PLANE GRATING
THE TESTING AND SELECTION OF STANDARDS[1]
SECOND PAPER

By CHARLES E. ST. JOHN AND L. W. WARE

APPARATUS AND METHOD

In this second paper we have examined the international secondary standards from λ 4282 to λ 5506 as to their consistency among themselves, and have determined the wave-lengths in international units of a series of 198 lines in the arc spectrum of iron from λ 4118 to λ 5506. The region from λ 5371 to λ 5506 is common to the 1912 and 1913 investigations, but an entirely new series of plates was made for the common region. The Pasadena plates were taken by Mr. Babcock and one of the writers. The spectrograph used was the 30-foot instrument in the Pasadena laboratory, which had been completely readjusted by Mr. Babcock after an examination and testing of the conditions requisite to the best performance. The grating was ruled by Anderson and is of the highest quality—a plane grating having a ruled surface 64×73 mm with 590 lines to the millimeter. The performance of the instrument is most satisfactory. The diffraction pattern in the case of the sharp lines is perfectly symmetrical on the two sides of the line with the first minimum absolutely black, by both visual and photographic tests. Each portion of the ruled surface is consistent with any other portion, so that errors arising from varying illumination are reduced to a minimum. The series of Mount Wilson plates was made with the 75-foot Littrow spectrograph used in conjunction with the 150-foot tower telescope. The grating (by Michelson) has an available ruled surface 57×124 mm and gives excellent definition for both bright and dark lines. In both series the Pfund arc[2] was used upon a 110-volt direct-current circuit. The length of the arc was 5–6 mm. An enlarged image was thrown upon the slit which was placed transverse to the axis

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 75.

[2] *Astrophysical Journal*, **27**, 296, 1908.

of the arc near its middle point. The focal lengths of the pro-
jecting lenses were such that the gratings were always filled. The
grating in the 75-foot spectrograph was in the axis of a light cone
1.5 m in diameter at the grating level. The current was main-
tained at approximately 6 amperes. The scale of the Pasadena
plates, second order, is 1 mm=0.88 A; of the Mount Wilson
plates, first order, 1 mm=0.72 A. They were measured either
upon a Toepfer comparator with a screw 300 mm long or upon a
comparator constructed in the Observatory instrument shop with
which a plate 500 mm long can be measured without readjustment.
The periodic errors of the screws are negligible. The errors of
run have been determined and the necessary corrections have
been applied to the scale readings. The plates have been measured
red right and red left. At least four settings have been made on
every line in the two positions, and in the case of the international
secondaries, the settings have frequently been doubled, particularly
when these standards have offered more than usual difficulties in
the determination of their position. The comparator microscopes
are equipped with single and double cross-wires. That used de-
pends upon the character of the line under consideration; frequently
settings have been made by both methods and the means taken of
the closely agreeing readings. A gain in convenience of measure-
ment and in accuracy was made by shading the strong lines on the
laboratory plates during a part of the exposure so that all the lines
were of more nearly uniform intensity. This proved particularly
helpful with lines that have a tendency to widen unsymmetrically,
and in the case of very strong lines the edges of which become more
or less fringed when the exposure time is such as to bring out the
weaker lines.

The method of reduction was fully described in our first paper.[1]
In brief, the reduction factors obtained by dividing the intervals
in angstroms between each two successive international standards
by the measured distance between the lines were plotted as ordi-
nates with the mean wave-lengths of the corresponding intervals
as abscissae, and as smooth a curve as possible was passed through

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 61; *Astrophysical Journal*, **36**, 19, 1912.

the points. The smoothness of the curves has been used as a test of the consistency of the international secondaries. In the case of standards 50 angstroms apart a change of 0.002 A in the wave-length of one of them produces a marked irregularity in the curve. The accuracy can be pushed to a high degree by repeated measurements of the plates. We have combined the results from different plates to separate what was accidental from what was constant in the course of the curves. In the series of overlapping plates, a line has in general occurred at three different positions so that the standards have been combined in various ways and are interlocked throughout the region investigated. The plates are 43 and 60 cm long, so that with the scales employed a number of standards occur on each plate, amply sufficient to determine the course of the curve, which in each case is nearly linear. Each line has been referred to both the standards between which it stands, but the course of the curve at the points from which the factors are taken for the reduction depends upon four standards so that the wave-length is intimately interlocked with the standards.

THE SELECTION OF STANDARD LINES

In the discussion of our results it will appear that they are closely related to the classification of the arc lines of iron proposed by Gale and Adams[1] and based upon the behavior of the lines under pressure.

To the five classes determined by the characteristics of the lines we have added a sixth:

1. Lines which are symmetrically reversed.
2. Lines which are unsymmetrically reversed.
3. Lines which remain bright and fairly narrow under pressure.
4. Lines which remain bright and symmetrical but become wide and diffuse under pressure.
5. Lines which remain bright and are widened very unsymmetrically toward the red.
6. Lines which remain bright and are widened very unsymmetrically toward the violet.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 58; *Astrophysical Journal*, **35**, 10, 1912.

Gale and Adams suggested a grouping of these lines according to pressure-shift, as follows:

Group a.—This includes the flame lines; these show small pressure displacements.

Group b.—This is a large group including all lines of moderate displacement and is probably complex.

Group c.—This consists of lines showing much larger displacements than those of group *b* and includes two fairly distinct classes.

Group d.—This is made up of lines showing immense displacements and unsymmetrical widening toward the red under pressure. We were led to divide group *d*, calling the separated portion *sub-d*. These lines are similar in behavior but show much smaller displacements and much less dissymmetry than those assigned to *d*.

Group e.—To the above groups we have added a group consisting of lines that are greatly displaced and unsymmetrically widened toward the violet under pressure.

The pressure-shift of the iron lines is still under examination by Gale and Adams. We have had access to the preliminary results of this investigation and so have been able to assign more lines to their probable groups.

The lines that remain symmetrical and reverse symmetrically under pressure are best fitted for standards; such are the lines of groups *a* and *b* which belong to classes 1, 3, and 4. The international standards of the second order from λ 4282 to λ 4531, with the exception of λ 4494, belong to these two groups and are lines that can be measured with high precision. λ 4494 and probably the secondaries λ 4547 to λ 4789 belong to group *c*, class 4, but from λ 4859 to λ 5001 they belong to group *c*, class 5. The lines of group *c*, class 4, are next in quality to those of groups *a* and *b*. Those of group *c*, class 5, are troublesome as standards. It is difficult to obtain consistent results with them, as they have a tendency to asymmetry toward the red. From λ 5012 to λ 5167 the lines belong to group *a* and are of excellent quality but from λ 5192 to λ 5324 they are of class 5 and belong to group *d* or *sub-d*, probably the latter. These are lines that are still more troublesome than those of group *c*, class 5. The selection of these lines for secondary standards was particularly unfortunate, as lines of group *a* of

excellent quality that could replace them occur in this region, as follows:

Present secondaries: group *sub-d*, λ 5192, λ 5232, λ 5266, λ 5302, λ 5324

Suggested secondaries: group *a*, λ 5194, λ 5216, λ 5227, λ 5270, λ 5328, λ 5341

The only space that cannot be filled by *a* lines is that occupied by λ 5302, but this line is so weak that it has been very difficult to obtain it strong enough for measurement without overexposing the other lines.   In fact, we have not used it as a standard but have determined its wave-length as for an unknown line.   The gain in having dependable standards of the second order would far outweigh any lack in desirable spacing.   From λ 5371 to λ 5506 the international standards belong to group *a* and are again of excellent quality.   From λ 5569 to λ 5763 they are of class 5 and belong to group *sub-d*.   These lines cannot be depended upon when the highest precision is required and should always be employed with caution.   From λ 6027 to λ 6494 the secondaries are all of group *b*, and with these as with those of group *a*, a precision of 0.001 A can be obtained.   Fortunately no lines of groups *d* or *e* occur among the secondaries.   These, in our judgment, based upon considerable experience, are entirely unsuitable for standards of any order, and happily they can be omitted without serious inconvenience.

It is of vital importance for standard wave-length determinations to fix the cause of large discrepancies.[1]   From his remarks at the meeting of the British Association at Dundee,[2] one must judge that Professor Kayser is somewhat discouraged with the outcome of the work of wave-length determinations.   He had hoped a few years ago, he said, to establish the third decimal place. He now finds uncertainties in the second decimal, depending upon the part of the arc used and other obscure causes.   Goos[3] raises the question of producing a light-source suitable for obtaining consistent results.   He used the interferometer method for checking

[1] The effects of varying conditions on the arc and of using its different parts are now being investigated in the observatory laboratory.

[2] *The Observatory*, **35**, 383, 1912.

[3] *Astrophysical Journal*, **37**, 48, 1913.

some results obtained with the plane grating. He found that with a short arc, 3 to 4 mm long, certain lines gave very poor or no interference phenomena, but with the middle of a 10 mm arc, using the same current, it was possible to obtain fair interference rings for these lines. He went to Kayser's laboratory in Bonn, where he again tried the short and long arc with the concave grating and found for certain lines a marked difference between the two light-sources. Now it is interesting to notice that all these lines for which he found differences in the two sources were shorter in the denser arc, and, so far as they are common to our lists, belong to group *e*. The characteristics of the lines of this group are displacements to the violet under pressure and great unsymmetrical widening on the violet edge when the pressure or density of t. e emitting vapor is increased. In the short arc, carrying the same current as the long arc, the density and possibly the pressure also in the central portion would be the greater and the wave-length would be shortened from one or both causes.

In writing the first paper based upon the Mount Wilson investigations, in a paragraph omitted because it was in a portion reserved for a later contribution but given in substanc in a paper read at the Cleveland meeting of the American Physical Society,[1] it was said:

An increase of intensity and consequent unsymmetrical widening may be an insidious cause of error in measuring such lines. This was clearly shown when the results of our measurements were finally assembled. Upon Plate 539 taken on the mountain there are five exposures of equal length. This plate was measured once by each of us and again unintentionally by one of us. It was noticed when the measurements were finally assembled that the wavelengths of λ 6003 and λ 6008 were longer for exposure 3 than for any of the other exposures. Upon the plate, besides the lines belonging to group *d*, displaced and widened to the red under pressure, were lines of group *e*, displaced and widened to the violet under pressure, and lines belonging to the same group as the standards, namely group *b*.

Table I shows the differences obtained by subtracting the mean of the values given by the four other exposures from the corresponding values obtained from the third exposure.

The systematic differences are striking, and at first were very misleading.

[1] "Standards of Wave-Length and Desirable Data for Them and for Other Lines." *Physical Review*, **1**, 67, 1913.

At the time the plate was measured we did not know of the existence of a group with the characteristics of group *e*.   It was only after a long wandering in the dark that we convinced ourselves that the differences in the measurement in lines belonging to groups *d* and *e* were inherent in the characteristics of the lines, and that the differences between their wave-lengths on Mount Wilson and in Pasadena were due to the enormous pressure-shifts that these lines undergo.   There was nothing in the appearance of Plate 539 to put us on our guard, and this is particularly true when a solar comparison is on each side of the narrow arc spectrum on this plate.   Exposure 3, however, is stronger than the others owing to the increased production of vapor in the arc during that exposure, and it may be a question whether the displacements were the effects of unsymmetrical broadening due to increased density or of a temporary increase in pressure due to a rapid evolution of vapor.   In the case of lines so sensitive to pressure changes as those of groups *d* and *e*, it may be a question whether rapid changes in the rate of vapor production in the arc may not cause temporary changes in pressure sufficient to displace these lines appreciably.   A mean change of pressure of one twenty-fifth of an atmosphere during the short exposure could produce a measurable effect.

TABLE I

EFFECT OF VARYING DENSITY IN THE ARC UPON LINES OF GROUPS *d* AND *e*

Group *b*		Group *d*		Group *e*	
λ 6020	+0.001 A	λ 5983	+0.006 A	λ 5984	−0.009 A
λ 6127	− .001	λ 6003	+ .016	λ 5987	− .003
λ 6136	− .001	λ 6008	+ .017	λ 6042	− .006
λ 6151	.000	λ 6021	+0.011	λ 6055	− .011
λ 6157	−0.002			λ 6078	−0.007
Mean....	−0.0006 A		+0.012 A		−0.007 A

As has been mentioned, the effect of varying arc conditions is now being investigated in the Pasadena laboratory.   It can be stated from the preliminary examination that, if a difference of pressure exists in the arc, it is not of a magnitude sufficient to account for the large discrepancies in the case of lines of groups *d* and *e*.   The main cause of these differences is found, we believe, in the unsymmetrical widening of these lines as the negative pole is approached.   Pressure-shift *per se* is not troublesome when once it has been determined, but the great sensitiveness of these lines to density changes is, in our opinion, the seat of the difficulty. The integrating action of the concave grating due to its astigmatism

in the usual mounting spreads the polar dissymmetry over the whole length of the line and would affect the measurement of such lines to a high degree.   We would recommend an arc 5–6 mm long, carrying a current of 5–7 amperes, and preferably the Pfund form because of its great convenience and steadiness of action; that an image enlarged at least two to three times be used, and that the slit be placed at right angles to the axis of the arc at the middle point of the image.   The lines with a short slit then have good edges and uniform widths throughout their length and good lines are capable of very exact determination.   Under these conditions by keeping the lines of moderate intensity, quite consistent results can be obtained even in the case of lines of group c, class 5, and of group sub-d; that is, one can expect a precision of 0.002 to 0.003 A. In the case of lines of groups a, b, and c 4, the ordinary conditions of the arc are in great probability without measurable influence.

RESULTS

The general results of our examination of the secondary standards and our measurements of the tertiary standards from λ 4118 to λ 5371 are shown in Table II.

The lines, as far as possible, have been grouped and classed according to the scheme proposed by Gale and Adams, with the subdivision of group d and the addition of group e.   In forming the means for the eighth column the mountain value of lines of groups d, sub-d, and c 5, when referred to the standards of groups a and b, have been given a positive correction of from 0.001 to 0.005 A, and the mountain values for the lines of group a, referred to standards of groups c and sub-d, have been decreased by 0.003 A in the formation of the means.   To allow for the pressure difference of one-fifth of an atmosphere between Pasadena and Mount Wilson the value of the correction is taken from the published and unpublished determinations of Gale and Adams and from the correction which we were obliged to apply in order to make the mountain standards consistent with the Pasadena values when standards of different groups were used together.   In the case of lines of group e, only the Pasadena values are used in comparison with Kayser and Goos as indicated by "P" in the last column.   In this column

we have also given some indications of the degree of dependence that can be placed upon the results obtained from the use of the lines as standards. In regions where secondary standards of groups *a*, *b*, and *c* 4 are available, we would place little reliance upon the lines indicated as unreliable and would not employ them at all if a high accuracy were desired. If only a fair precision is required they will be useful, if care is taken to have these lines of moderate intensity. This is best done by using only the middle of the arc. The difficulties inherent in the lines of groups *d* and *sub-d* increase with the wave-length; those of group *d* are practically unusable below λ 5000. For the region of shorter wave-length, no clear distinction can, as yet, be made between groups *d* and *sub-d*. All lines of group *e* are, according to our experience, unusable as standards. When a line is so weak upon our plates that the determination is not of high weight, we have indicated the fact by the word "faint." The figures in parentheses give the number of times the lines have been measured.

DISCUSSION

In Table III are assembled the wave-lengths of all the standards of the second order for which a deviation from the international value exceeding 0.002 A has been found by any observer. The second column contains the maximum deviations from the accepted value in the case of the th ee independent determinations of the standards by the interferometer method. In the three succeeding columns are the deviations found by the three observers using the spectrographic method. Our results seem to indicate a remarkable degree of consistency among the international standards, and we are confident that the method we have employed when pushed to its limiting accuracy is capable of detecting deviations of 0.002 A in the relative wave-lengths of a series of standards. The average deviation for the interferometer method is 0.0015 A, and that shown by Kayser's results is 0.0045 A. It seems to us so improbable that errors of this latter magnitude can exist in the secondary standards and escape detection by the method we have employed that we are forced to the conclusion that some obscure source of error affects the determinations published by Professor Kayser. The deviations from the accepted international values exceeding

TABLE II

Group	Class	International Wave-Length Pasadena	International Wave-Length Mt. Wilson	Mean Deviation P.	Mean Deviation Mt.	P.–Mt.	Mean Wave-Length	M.–L.	M.–K.	M.–G.	Remarks
		(4118.553) (11)							(000)		From λ4134 to λ4531, except λ4404, the lines used as secondary standards belong to groups *a* and *b* and are first-class in quality
b	1	4121.809 (11)		2			4121.809 (11)		+0.002		
		4127.615 (11)		1			4127.615 (11)		.000		
		4132.062 (11)		2			4132.062 (11)		−.002		
b	4	4134.685									
		4137.002 (11)		2			4137.002 (11)		+.001		
		4143.422 (11)		1			4143.422 (11)		+.001		
b	1	4143.872 (11)		2			4143.872 (11)		+.002		
		4147.676									
		4153.914 (4)	.917 (7)	1	3	−0.003	4153.916 (11)		−.001		
		4154.505 (5)	.504 (9)	1	1	+.001	4154.505 (20)				
		4154.815 (4)	.816 (8)	1	2	−.001	4154.816 (12)		−.001		
		4156.805 (11)	.805 (10)	1	1	.000	4156.805 (21)				
		4157.804 (7)	.806 (9)	5	4	+.002	4157.805 (16)				
		4170.906 (17)	.907 (7)	3	1	−.001	4170.906 (24)		.000		
		4172.126 (4)	.126 (5)	1	2	.000	4172.126 (9)				
		4174.918 (1)	.917 (5)	0	2	+.001	4174.917 (6)				
		4175.639 (17)	.640 (10)	3	2	−.003	4175.639 (27)		.003		
		4176.563 (5)	.566 (7)	4	4	−.003	4176.565 (12)				
		4177.595 (5)	.598 (6)	4	2	−.003	4177.597 (11)				
		4181.758 (17)	.757 (9)	3	2	−.001	4181.758 (26)				
		4184.895 (17)	.895 (10)	4	1	.000	4184.895 (27)		+.001		
		4187.051 (17)	.049 (9)	2	2	+.002	4187.050 (20)		−.000		
		4187.806 (8)	.805 (9)	2	2	+.001	4187.805 (17)		−.002		
		4191.441		3	1	−.008			−.000		
e		4191.671 (7)	.679 (6)	0	1	−.002	4195.343 (11)		−.006		
		4195.342 (4)	.344 (11)	1	2	+.001	4198.307 (22)		.000		
b	1	4198.306 (11)	.307 (11)	3	1	+.001	4199.099 (26)		.000		P. Unusable. Faint
		4199.099 (14)	.008 (12)	3	2	−0.001	4202.031 (26)		+0.001		
		4202.031 (14)	.032 (12)								

						Δ			Δ	Δ	Remarks
b	3	4203.988 (4)	.086 (8)	0	3	+0.002	4203.987 (12)		+0.002		Unreliable. Edges hazy. Mt. +0.002
c	5	4210.358 (11)	.360 (12)	1	2	.002	4210.359 (23)		+.002		Unreliable. Mt. +0.002
b	3	4216.187 (14)	.188 (11)	3	3	.001	4216.187 (25)				Unreliable. Mt. +0.002
		4217.558 (2)	.561 (8)	1	1	.003	4217.560 (10)		+.000		Unreliable. Probably group e
		4219.365 (14)	.366 (12)	2	3	.001	4219.365 (26)		+.004		Mt. +0.001
		4222.223 (11)	.225 (12)	2	3	.002	4222.224 (23)				
		4224.177 (4)	.177 (8)	1	2	.000	4224.177 (12)		+.007		
		4225.406 (4)	.409 (8)	2	2	+.003	4225.408 (12)		+.005		
d	5	4227.446 (11)	.445 (12)	7	8	.001	4227.447 (23)				
d	5	4233.613 (11)	.615 (13)	2	6	.002	4233.615 (24)		+.004		Measurements difficult
d	5	4235.948 (11)	.950 (12)	3	7	.002	4235.950 (23)		+.004		Difficult of exact determination
		4247.439 (15)	.444 (14)	2	6	.005	4247.441 (29)		+.003		Reversed .765. Unreversed .764. Always good
c	5	4250.132 (17)	.133 (12)	3	7	.001	4250.133 (29)		+.001		
b	2	4250.701 (17)	.793 (12)	2	7	.002	4250.792 (29)		+.005		
c	2	4260.486 (17)	.484 (12)	4	8	+.002	4260.485 (29)		+.004		
		4267.833 (8)	.833 (6)	3	1	+.000	4267.833 (14)		+.019		
b	1	4271.168 (17)	.166 (12)	6	5	+.001	4271.167 (29)				Unreliable. Mt. +0.002
		4271.765 (11)	.764 (12)	1	2		4271.764 (23)	0.000	+.004	0.000	
b	1	4282.408		1	1	+.000			+.007	+.001	
b	2	4294.131 (11)	.131 (26)	5	4	.002	4294.131 (37)		+.007	+.003	Reversed .909. Unreversed .907. Difficult when unreversed
d	5	4299.250 (11)	.248 (27)				4299.250 (38)				
b	1	4307.906 (14)	.909 (23)	3	3	−0.003	4307.908 (37)		−.037	−.004	
b	3	4315.090						+0.001	+0.001	+0.001	

TABLE II—*Continued*

Group	Class	International Wave-Length Pasadena	International Wave-Length Mt. Wilson	Mean Deviation P.	Mean Deviation Mt.	P.—Mt.	Mean Wave-Length	M.—I.	M.—K.	M.—G.	Remarks
b	I	4325.766 (14)	.766 (27)	4	4	0.000	4325.766 (41)		+0.003	+0.002	Reversed .768. Unreversed .765. Difficult when unreversed
b	3	4337.051 (11)	.052 (26)	I	I	− .001	4337.052 (37)		− .001	.000	Excellent line
b	3	4352.739						−0.002	− .001	.002	
b	3	4367.581 (2)	.583 (7)	4	I	− .002	4367.582 (9)	.000	+ .009	.002	
	3	4369.777 (11)	.775 (19)	I	I	+ .002	4369.776 (30)		+ .004	.002	
a	3	4375.934									
b	I	4383.548 (14)	.549 (27)	3	4	− .001	4383.549 (41)		− .003	.002	Reversed .551. Unreversed .547. Difficult when unreversed
b	I	4388.414 (I)	.412 (I)	0	0	+ .002	4388.413 (2)		+ .001	.007	
		4404.754 (14)	.754 (26)	2	3	.000	4404.754 (40)			.001	Reversed .756. Unreversed .753. Difficult when unreversed
c	4	4407.714 (15)	.717 (7)	3	2	− .003	4407.715 (22)			.001	Faint
c	4	4408.420 (10)	.420 (12)	I	2	.000	4408.420 (22)			.000	
b	I	4415.128 (11)	.127 (26)	2	2	+ .001	4415.127 (37)	0.000	+ .002	.002	Unreversed. Good line
b	3	4422.572 (11)	.572 (16)	2	2	.000	4422.572 (27)		+ .010	.000	
a	3	4427.314							.000	.000	
c	4	4430.622 (5)	.620 (13)	I	2	+ .002	4430.621 (18)		+ .003	.002	
c	4	4442.345 (5)	.344 (13)	0	I	+ .001	4442.344 (18)		+ .001	.005	
c	3	4443.198 (5)	.196 (13)	I	2	+ .002	4443.197 (18)		.000	.002	
c	4	4447.724 (5)	.723 (14)	I	I	+ .001	4447.723 (19)		.000	.002	
b	3	4454.387 (5)	.384 (8)	0	2	+ .003	4454.385 (13)		+ .003	+ .001	
c	4	4459.124 (5)	.123 (12)	I	2	+0.001	4459.123 (17)		+0.003	−0.004	

		λ				±	λ		±	Remarks
a	3	4461.657 (5)	.655 (14)	1	2	+0.002	4461.656 (19)	0.000	+0.004	Unreliable. Hazy on red edge
b	4	4466.556 (7)							+.003	Measures influenced by closeness
b	4	4469.386 (7)	.382 (4)	1	2	+.004	4469.385 (11)		+.003	
		4476.025 (7)	.027 (9)	6	1	-.002	4476.026 (16)		+.002	
		4482.164 (6)	.173 (5)	2	3	-.009	4482.168 (11)		-.005	
a	3	4482.270 (6)	.260 (6)	3	2	+.010	4482.265 (12)	.000	-.001	
		4484.236 (7)	.235 (1)	1	0	+.001	4484.236 (8)		+.002	
		4489.746 (7)	.744 (1)	1	0	+.002	4489.746 (8)		+.001	
c	4	4490.090 (7)		1			4490.090 (7)		-.001	
		4494.572							-.004	
		4514.192 (2)	.192 (2)	2	0	.000	4514.192 (4)		+.001	
c	4	4517.532 (2)	.535 (4)	0	1	-.003	4517.534 (6)	.000	-.009	
		4525.150 (7)	.151 (8)	2	2	-.001	4525.151 (15)		-.004	
c	4	4528.620 (8)	.623 (16)	1	2	-.003	4528.622 (24)		+.000	
b	3	4531.155							+.001	Faint
		4547.853							-.002	P. Unusable. Hazy
e		4552.551 (1)	.548 (1)	0	0	+.003	4552.550 (2)	.000	-.003	Faint
		4556.128 (7)	.131 (12)	3	3	-.003			-.005	
		4574.726 (1)	.727 (3)	0	0	-.001	4574.727 (4)		-.019	
		4581.522 (2)	.522 (4)	4	1	.000	4581.522 (6)		-.002	
		4592.658							-.013	Faint
		4598.132 (4)	.130 (5)	2	1	+.002	4598.131 (9)	.000	+.009	Faint
		4602.008 (5)	.009 (4)	1	1	+.001	4602.008 (9)		-.002	From λ4547 to λ4789 except λ4707 and λ4736 secondaries belong probably to group c 4; second-class in quality
		4602.947								
		4607.663 (7)		2	2		4607.663 (7)		+.001	
		4611.292 (7)	.292 (6)	1		.000	4611.292 (13)	0.000	-.001	
		4613.219 (3)	.217 (5)	4	2	+.002	4613.218 (8)		-.018	Faint
d?		4619.297 (7)	.208 (5)	2	3	-.001	4619.297 (12)		-.003	Unreliable. Hazy and faint
		4625.062 (7)	.056 (6)	3	4	-.006	4625.059 (13)		+.003	
		4630.130 (7)	.128 (5)	2	2	+0.002	4630.129 (12)		-0.014	

TABLE II—Continued

Group	Class	International Wave-Length Pasadena	International Wave-Length Mt. Wilson	Mean Deviation P.	Mean Deviation Mt.	P. − Mt.	Mean Wave-Length	M. − I.	M. − K.	M. − G.	Remarks
d?		4637.522 (7)	.515 (6)	3	2	+0.007	4637.519 (13)		+0.002	−0.009	Unreliable
		4638.018 (7)	.017 (6)	2	4	+ .001	4638.018 (13)		− .001	− .008	
		4647.439								+ .000	
		4654.503 (7)	.503 (4)	1	1	+ .000	4654.503 (11)	0.000	+ .003	− .011	⎫ Measures influenced
		4654.638 (7)	.630 (4)	2	1	+ .008	4654.635 (11)		+ .008	− .010	⎭ by closeness
		4667.459 (7)	.462 (6)	2	3	− .003	4667.460 (13)		− .004	+ .003	
d?	4	4668.151 (7)	.147 (6)	2	2	+ .004	4668.149 (13)		+ .003	− .008	Unreliable
	4	4673.169 (6)	.171 (4)	3	2	− .002	4673.170 (10)		+ .001	+ .008	Faint
	5	4678.857 (7)	.857 (6)	1	3	+ .000	4678.857 (13)		+ .003	− .001	
		4691.417								+ .003	
		4707.288								+ .002	
d?		4710.287 (8)	.288 (9)	2	4	− .001	4710.288 (17)		− .003	− .001	Unreliable
		4727.438 (8)	.432 (7)	4	4	+ .006	4727.435 (15)	.000	+ .002	+ .007	
		4733.594 (13)	.598 (8)	3	2	− .004	4733.596 (21)	.000	− .001	+ .010	
		4736.786								+ .009	
c	5	4741.532 (13)	.534 (6)	1	3	− .002	4741.533 (19)		+ .003	− .003	
		4745.806 (11)	.807 (5)	3	4	− .001	4745.806 (16)		+ .004	+ .004	Faint
		4754.048 (14)	.045 (9)	2	2	+ .003	4754.047 (23)	.000	− .002	+ .014	
		4757.582 (2)	.582 (1)	4	0	+ .000	4757.582 (3)		+ .006	+ .001	
		4762.375 (14)	.374 (8)	1	3	+ .001	4762.375 (22)		− .004	+ .010	Faint
	5	4765.865 (13)	.864 (6)	2	2	+ .001	4765.865 (19)		+ .003	− .003	Faint
c		4766.425 (13)	.426 (7)	2	1	− .001	4766.425 (20)		+ .002	+ .004	
c		4772.818 (12)	.818 (5)	2	1	+ .000	4772.818 (17)		− .003	+ .005	
c	5	4783.435 (14)	.431 (9)	3	2	+ .004	4783.433 (23)		+ .002	− .006	
c	4	4786.813 (13)	.812 (7)	1	2	+ .001	4786.813 (20)		+ .006	+ .004	
c	4	4788.764 (4)	.764 (2)	1	2	+ .000	4788.764 (6)		+ .001	+ .003	
c	4	4789.657								− .002	
c	4	4823.527 (7)	.522 (9)	2	2	+ .005	4823.524 (16)	.000	+ .007	+ .003	
c	5	4839.552 (2)	.553 (4)	2	1	−0.001	4839.553 (6)	0.000	+0.004	+ .002	
c	5	4859.758								+0.002	

Gr.	5	λ		n	n′	Δ		λ	δ	δ	Notes
c	5	4871.331 (7)	.333 (11)	3	2	−0.002	0.000	4871.332 (18)	−0.001	+0.003	From λ4859 to λ5001 the secondaries belong to group c, and are third-class in quality
c	5	4872.152 (6)	.152 (11)	4	2	.000		4872.152 (17)	.003	.003	
c	5	4878.225							.004	.001	
		4881.725 (1)	.724 (2)	0	2	+.001		4881.724 (3)	.000	.006	
c	5	4885.440 (1)	.443 (2)	0	1	.003		4885.442 (3)	.003	.001	
c	5	4890.767 (7)	.769 (11)	3	1	.002		4890.768 (18)	.003	.008	
c	5	4891.504 (7)	.506 (11)	4	2	.002	.000	4891.505 (18)	.005	.005	
		4903.325							.002	.000	Mt. −0.003
c	5	4910.036 (4)	.036 (3)	3	3	.000	.000	4910.036 (7)	.005	.001	
		4919.007							.007	.001	Mt. −0.003
c	5	4920.517 (15)	.514 (11)	3	2	+.003		4920.516 (26)	.007	.002	
c	5	4924.775 (6)	.779 (6)	3	3	.004		4924.776 (12)	.001	.008	
c		4938.827 (15)	.827 (11)	3	1	.000	.000	4938.827 (26)	.003	.000	
c		4939.688 (19)	.691 (21)	3	1	.003		4939.688 (40)	.001	.009	
a		4946.404 (10)	.403 (5)	2	2	.001		4946.404 (15)	.004	.002	
		4957.310 (14)	.390 (11)	3	3	.001		4957.310 (25)	.007	.004	
		4957.609 (15)	.668 (11)	3	3	.001		4957.609 (26)	.000	.004	
		4966.104							.001	.001	
d		4973.115 (3)	.114 (4)	2	3	+.001		4973.114 (7)	.005	.004	Mt. +0.005
c		4978.614 (1)	.616 (2)	0	2	.002		4978.615 (3)	.003	.009	} Apparently uninfluenced by closeness
c		4982.532 (12)	.528 (8)	5	4	.004	+.001	4982.532 (20)	.006	.001	
c		4985.268 (12)	.269 (7)	1	2	.004		4985.268 (21)	.008	.003	Mt. −0.004
a		4985.560 (14)	.564 (7)	1	2	.004		4985.561 (21)	.005	.001	
c		4994.133 (19)	.138 (22)	3	2	.005		4994.134 (41)	.003	.001	
c		5001.881							.002	.002	
c		5005.730 (15)	.726 (16)	2	3	.004		5005.730 (31)	.001	.002	Mt. +0.004
a		5006.135 (15)	.131 (17)	3	3	.004		5006.135 (32)	.004	.002	Mt. +0.004
c		5012.074							.003	.000	
c		5014.960 (11)	.958 (12)	3	3	.002		5014.961 (23)	.005	.004	Mt. +0.004
a		5022.253 (9)	.249 (9)	3	3	.004		5022.253 (18)	.006	.004	Mt. +0.004 Faint
a		5028.131 (1)	.129 (2)	0	0	.002		5028.130 (3)	.002	.004	
a		5041.077 (4)	.077 (9)	2	2	.000		5041.077 (13)	.002	.005	
a		5041.761 (8)	.760 (12)	1	3	.001		5041.760 (20)	.001	.002	
a		5049.827							.001	.001	
a		5051.639 (8)	.640 (6)	2	2	−0.001	0.000	5051.639 (14)	−0.001	−0.001	

TABLE II—Continued

Group	Class	International Wave-Length Pasadena	International Wave-Length Mt. Wilson	Mean Deviation P.	Mean Deviation Mt.	P.—Mt.	Mean Wave-Length	M.—I.	M.—K.	M.—G.	Remarks
c		5068.783 (4)	.778 (6)	3	4	+0.005	5068.782 (10)		−.004	−0.004	Mt. +0.004
e		5074.739 (6)	.741 (9)	4	5	−.002			−.001	+.017	P. Unusable. Hazy
a		5079.229 (8)	.226 (6)	2	5	+.003	5079.228 (14)		−.003	+.002	From λ5012 to λ5167
a		5079.745 (8)	.741 (5)	1	2	+.004	5079.743 (13)		−.008	+.002	the secondaries belong to group a and are first-class in
a		5083.344									quality
a		5098.708 (13)	.704 (22)	2	2	+.004	5098.705 (35)	0.000	−.002	+.009	Measures influenced
a		5105.546 (2)	.544 (3)	2	3	+.002	5105.545 (5)		+	−.002	by closeness
a		5107.454 (8)	.455 (3)	2	3	−.001	5107.454 (14)		−.008	−.014	
a		5107.647 (8)	.646 (6)	3	4	+.001	5107.647 (14)		−.003	−.004	
a		5110.415									
a		5123.727 (8)	.729 (6)	1	2	−.002	5123.728 (14)		+.003	+.004	
a		5127.366 (8)	.370 (6)	2	2	−.004	5127.368 (14)		−.001	+.001	
e		5133.656 (9)	.674 (10)	7	5	−.018		.000	+.019	+.010	P. Unusable. Very hazy
c		5139.269 (8)	.270 (6)	3	3	−.001	5139.271 (14)		+.003	−.007	Mt. +0.003. Apparently uninfluenced
c		5139.484 (8)	.481 (6)	4	6	+.003	5139.484 (14)		+.002	−.009	by closeness. Mt. +0.003
.		5142.933 (8)	.937 (6)	1	2	−.004	5142.935 (14)		+.001	+.001	
a		5150.845 (14)	.849 (8)	2	3	−.004	5150.846 (22)		+.005	−.001	
d		5151.916 (8)	.920 (5)	2	3	−.004	5151.918 (13)		+.003	+.002	
a		5162.336 (4)	.319 (9)	3	7	+.017		.000	+.026	−.011	P. Unusable. Hazy
		5167.492									
a		5168.904 (4)	.905 (3)	0	1	−.001	5168.904 (7)		+.004	+.002	
a		5171.600 (8)	.603 (11)	2	3	−.003	5171.602 (19)		−.003	.000	Mt. +0.004
c sub-d		5191.474 (19)	.470 (10)	2	1	+.004	5191.474 (29)	0.000	+.003	−.005	
		5192.363									
a		5194.949 (18)	.951 (9)	4	2	−.002	5194.949 (27)		+.003	+.002	Mt. +0.004
a		5198.719 (14)	.721 (4)	4	3	−.002	5198.719 (18)		−.002	−.001	Mt. −0.003
a		5202.341 (11)	.343 (17)	4	4	−0.002	5202.340 (28)		−0.001	−0.001	Mt. −0.003

Group	Class	λ (wt.)	frac. (wt.)	n	n	O−C	λ (wt.)		O−C	O−C	Remarks
sub-d		5208.612 (15)	.612 (5)	2	2	0.000	5208.612 (20)	0.000	−0.007	−0.002	
sub-d		5215.199 (17)	.201 (4)	3	1	−.002	5215.199 (21)		+.006	+.002	
c		5216.282 (19)	.283 (9)	5	2	−.001	5216.281 (28)		+.006	+.002	Mt. −0.003
c		5217.407 (15)	.410 (4)	2	3	+.003	5217.408 (19)		−.005	−.001	
c	4	5226.881 (17)	.879 (11)	1	2	+.002	5226.880 (28)		+.006	+.007	Mt. −0.003
sub-d	5	5227.193 (19)	.193 (11)	4	4	.000	5227.192 (30)		+.007	+.002	
		5229.863 (3)	.865 (4)	2	2	−.002	5229.864 (7)	.000	+.021	+.002	
		5232.957							−.003		
a		5242.497 (5)	.499 (6)	2	2	−.002	5242.496 (11)		+.001	+.001	Mt. −0.003. Faint
		5250.652 (7)	.652 (3)	2	2	+.000	5250.652 (10)	.000	.000	+.008	
sub-d		5263.320 (5)	.318 (4)	2	2	+.002	5263.319 (9)		.000	+.001	
sub-d		5266.569							+.005	−.008	
a		5269.541 (21)	.542 (33)	3	4	−.001	5269.540 (54)		+.005	+.005	Mt. −0.003. Measures difficult
a		5270.360 (11)	.359 (13)	3	3	+.001	5270.358 (24)	.000	+.004	+.001	Mt. −0.003
sub-d		5273.179 (6)		2			5273.179 (6)		+.003	+.011	
sub-d		5273.382 (6)		2			5273.382 (6)		−.004	−.004	
sub-d		5281.805 (11)	.803 (11)	2	2	+.002	5281.804 (22)		−.002	−.004	
sub-d		5283.636 (11)	.633 (13)	1	2	+.003	5283.634 (24)		−.001	−.001	Faint. Too weak for good secondary
sub-d		5302.318 (11)	.313 (22)	2	2	+.005	5302.315 (33)		+.001	+.001	
		5307.366 (4)		2			5307.366 (4)	.002	+.001	+.002	
		5324.194						−	−.003	−.001	
sub-d	5	5328.046 (11)	.043 (11)	3	3	+.003	5328.044 (22)		+.004	+.004	
a	1	5328.538 (11)	.536 (11)	2	2	+.002	5328.537 (22)		−.005	+.002	
a	4	5332.908 (6)		2			5332.908 (6)		−.004	−.004	
sub-d	4	5339.945 (10)	.942 (9)	2	2	+.003	5339.945 (19)		+.002	−.004	Mt. +0.003
a	5	5341.030 (11)	.028 (11)	2	4	+.002	5341.029 (22)		−.002	−.006	
e	4	5364.860 (7)	.862 (16)	7	4	−.002			+.002	+.009	
a		5365.407 (6)	.404 (6)	3	4	+.003	5365.405 (12)	0.000	−.001	+.001	P. Unusable. Hazy
a	1	5371.495							+0.005	−0.000	

NOTE.—From λ 5192 to λ 5324 the secondaries belong to group *sub-d* and are fourth-class in quality. For this region lines belonging to group *a* and first-class in quality are available for secondaries.

0.002 A shown by Goos's results are nearly all associated with lines that are inherently difficult of measurement, while in the Kayser series, the secondary standards of excellent quality, groups *a* and *b* show the same order of deviations as the poorer lines of groups *c* and *d*.

TABLE III

DEVIATIONS FROM THE INTERNATIONAL VALUES GREATER THAN 0.002 A

International Wave-Lengths	Maximum Deviation	I.—P.	I.—G.	I.—K.
4282.408	0.001	0.000	0.000	+0.004
4375.934	.001	.000	.000	+ .004
4466.556	.002	.000	+ .001	+ .003
4531.155	.000	.000	+ .001	− .004
4547.853	.001	.000	− .001	− .006
4647.439	.002	.000	.000	+ .003
4691.417	.002	.000	+ .003	+ .002
4736.786	.001	.000	− .003	− .001
4789.657	.001	.000	+ .003	+ .006
4859.758	.002	.000	+ .002	+ .004
4878.225	.001	.000	− .001	− .004
4919.007	.001	.000	+ .001	− .007
5001.881	.004	.000	− .001	− .003
5012.073	.001	− .001	+ .003	+ .003
5110.415	.001	.000	+ .004	+ .003
5167.492	.001	.000	+ .002	+ .004
5192.363	.001	.000	− .001	+ .001
5232.957	.001	.000	+ .002	− .003
5266.569	.001	.000	− .001	− .005
5371.495	.003	.000	.000	+ .005
5569.633	.003	− .001	.000	+ .003
5615.661	.003	.000	− .002	− .006
5658.836	.002	.000	+ .001	− .010
6027.059	.000	.000	+ .002	− .003
6065.492	.001	.000	.000	+ .003
6430.859	0.004	0.000	−0.001	+0.011
Sum of deviations	0.041	0.002	0.036	0.111
Mean deviation	0.0016	0.0001	0.0014	0.0043

PAIRS OF LINES

In fixing the standards, it becomes important to determine the exactness with which certain types of lines can be measured. They are of little use as standards if different observers set upon them differently, or if there is something in the character of the lines tending to produce a bias in the observer's mind or a conscious effort to avoid a possible error. In the lists of tertiary standards published there are nine pairs of lines with intervals from

TABLE IV

MEASUREMENT OF CLOSE PAIRS OF LINES

PASADENA		MT. WILSON		KAYSER		GOOS		MAX. RANGE		GROUP
λ	$\Delta r - \Delta v$	λ	$\Delta r - \Delta v$	λ	$\Delta r - \Delta v$	λ	$\Delta r - \Delta v$	λ	$\Delta r - \Delta v$	
4482.164	0.106	.173	0.087	.163	0.103	.173	0.097	0.010	0.019	
4482.270		.260		.266		.270		.010		
4654.503	.135	.503	.127	.495	.144	.514	.131	.019	.017	
4654.638		.630		.639		.645		.015		
4957.310	.299	.309	.299	.303	.306	.306	.307	.007	.008	c
4957.609		.608		.609		.613		.005		c
4985.268	.292	.269	.295	.274	.295	.269	.295	.006	.003	c
4985.560		.564		.569		.564		.009		c
5107.454	.193	.455	.191	.462	.188	.468	.175	.014	.018	a
5107.647		.646		.650		.643		.007		a
5139.269	.215	.270	0.211	.268	.214	.278	.215	.010	.004	c
5139.484		.481		.482		.493		.012		c
5273.179	.203			.176	.202	.178	.193	.003	.010	sub-d
5273.382				.378		.371		.011		sub-d
5462.957	.313			.964	.308	.952	.301	.012	.012	
5463.270				.272		.253		.019		
5476.297	0.286			.294	0.287	.300	0.286	.006	0.001	a
5476.583				.581		.586		0.005		d

0.1 to 0.3 A.   In determining the separation of such pairs, that is, in the measurement of such short intervals, neither the absence of complete normality in the spectra, nor the method of reduction, nor small errors in the reduction factor can have an appreciable effect. The agreement or the lack of agreement between the values obtained will indicate the degree of exactness with which settings can be made upon such lines by different observers, and will give a criterion for judging the fitness of such lines for standards. The data relative to these nine pairs are shown in Table IV.   In the third column from the end is given the range in the separate determinations of the wave-lengths.   This reaches very high values such as 0.012, 0.014, 0.015, and 0.019 A, much higher than for 18 lines taken at random.   The range in the intervals between the pairs is perhaps the most striking, attaining as it does a maximum of 10 per cent of the quantity measured, and a mean value of 0.010 A. It seems quite evident that the disturbing effect due to the presence of the near-by line is sufficient to render the settings upon both uncertain to a high degree.   This makes the exact determination of the wave-lengths extremely difficult, and seems to preclude their use in a series of standards aiming at an accuracy of 0.001 A. Even if their wave-lengths should be well determined from the means of a great number of measurements, their employment as standards would not be advisable because of the probability of a great error even in the means of a large number of settings by a single observer, dependent upon the dispersion, the cleanness of the lines, and the personal equation.

RELATIVE WAVE-LENGTHS PASADENA — MOUNT WILSON

Gale and Adams have shown that the pressure displacements of iron lines increase as the cube of the wave-length.   Therefore the changes in relative wave-length between Pasadena and Mount Wilson would be less in the case of the present series, λ 4118– λ 5371, than for the first, λ 5371–λ 6494, in a 1:2 ratio, and the effect due to a decrease of one-fifth of an atmosphere would be less in evidence.   A few lines of groups c and sub-d have been referred to standards of group a both in Pasadena and on Mount Wilson, and some lines of group a have been referred to standards of

groups *c* and *sub-d* in the two series. In the first instance the wave-lengths of lines of groups *c* and *sub-d* are relatively shortened at the higher elevation, and in the second instance the lines of group *a* are apparently lengthened on the mountain due to the lessened wave-lengths of the standards. The measurements are difficult in both cases, as in the first instance the lines to be measured are of poor quality, and in the second the reference lines are poor, though the lines to be measured are of high quality. The lines of group *e* will be displaced to the red under the lessened pressure at the higher elevation and hence the wave-lengths will be actually longer whatever secondary standards are used. The results in the three cases are shown in Table V.

TABLE V

CHANGE IN WAVE-LENGTH BETWEEN PASADENA (244 M) AND MOUNT WILSON (1794 M)

a LINES AGAINST *c* AND *sub-d* STANDARDS		*c* AND *sub-d* LINES AGAINST *a* STANDARDS		*e* LINES AGAINST *a, b,* AND *d* STANDARDS	
λ	P.—Mt.	λ	P.—Mt.	λ	P.—Mt.
4924	−0.004	5005	+0.004	4191	−0.008
4939	− .003	5006	+ .004	4556	− .003
4994	− .005	5014	+ .002	5074	− .002
5194	− .002	5022	+ .004	5133	− .018
5202	− .002	5068	+ .005	5364	−0.002
5216	− .001	5139	− .001		
5227	.000	5139	+ .003		
5242	− .002	5191	+ .004		
5269	− .001	5339	+0.003		
5270	+0.001				
Mean...	−0.0019		+0.0031		−0.007

REVERSED LINES

We have given in the last column of Table II the measurements made upon the reversals and upon the lines when unreversed. In the case of the strong lines $\lambda\lambda$ 4307, 4325, 4383, and 4404, the measurements upon the reversals are the more reliable, as these lines when not reversed are broad with edges often much fringed. The line λ 4271 is a good line under all conditions, and λ 4415 is an excellent line with sharp edges. It was not reversed upon our plates. These lines are referred to standards of good quality and if measured

as reversals any large variations could hardly be due to errors in the settings. The conspicuous differences between our results and Kayser's for $\lambda\lambda$ 4271 and 4307 we are unable to explain. An appeal to the Rowland wave-lengths gives no new light as the differences, Rowland−Kayser, for these two lines are still inconsistent with the other Rowland−Kayser differences. He gives a strong *Ca* line near λ 4307 which has never appeared upon our plates, and which if present at a distance of 0.073 A from the strong *Fe* line would make an exact determination impossible.

TABLE VI

COMPARISON OF THE WAVE-LENGTHS OF LINES WHEN REVERSED AND UNREVERSED

λ	REVERSED		UNREVERSED		WT. MEAN	MEAN−KAYSER	MEAN−GOOS	ROWLAND−MEAN	ROWLAND−KAYSER
	P.	Mt.	P.	Mt.					
4271..	.765	.766	.765	.763	.764	+0.019		0.170	0.189
4307..	.910	.909	.904	.909	.908	− .037	−0.004	.173	.136
4325..	.769	.767	.764	.766	.766	+ .003	+ .002	.173	.176
4383..	.551	.550	.546	.547	.549	− .003	− .002	.171	.168
4404..	.757	.755	.752	.754	.754	+ .001	− .001	.173	.174
4415..	....	....	.128	.127	.127	+0.002	−0.002	0.166	0.168

THE PRECISION OF THE MEASUREMENTS

The internal agreement of the series of 1912 was indicated by a mean probable error of 0.0007 A and 0.0006 A for the Pasadena and mountain plates, respectively, in the case of good lines. The two series of 1913 contain fewer plates but both are of better quality. particularly the Pasadena series. In the series of 1913 all lines —many of poor quality—have been included. The lines have been measured on the average 9 times; the mean probable errors for a single line are 0.0006 A and 0.0007 A, respectively, for the Pasadena and mountain series. If the lines of groups *d*, *sub-d*, and *e* were omitted, as in the case of the 1912 series, the probable errors would be less than for that series, even with the smaller number of measurements per line, as would be anticipated from the better quality of the plates. Other lines grouped about λ 4900 and belonging to group *c*, class 5, are accountable in part for the large residuals. In the case of these lines, as for those belonging to group *sub-d*, it is difficult to reach a precision greater than 0.002 A.

A short region, λ 5371–λ 5506, is common to the 1912 and 1913 series of determinations. It is of interest to see what degree of agreement, in the case of lines belonging to the better group *a*, may be obtained by the same observers working upon two different lots of plates. The second series of Pasadena plates was made with a new plane grating of the highest quality. The mountain plates were obtained with the 75-foot Littrow spectrograph of the 150-foot tower telescope. The comparison in shown in Table VII. The practically complete agreement between the two series in regard to both secondary and tertiary standards shows that the plane grating spectrograph of long focus can be considered an instrument of high precision and seems to justify its employment when the purpose is the interpolation between standards distributed through the spectrum, as are the international standards of the second order.

TABLE VII

STANDARDS COMMON TO THE TWO SERIES

1912	1913	1912 − 1913
5371.495	.495	0.000
5429.702	.702	.000
5434.529	.529	.000
5446.919	.921	− .002
5455.614	.614	.000
5462.957	.957	.000
5463.268	.270	− .002
5473.917	.915	+ .002
5476.296	.297	− .001
5497.522	.522	.000
5501.470	.471	− .001
5506.784	.784	0.000

SUMMARY

1. The results of our investigation of the secondary standards from λ 4282 to λ 5371 are as favorable as for the yellow-red region. Of the 32 standards of the second order in the blue-green region we find adjustments indicated in the case of four lines only, amounting to 0.001 A for two lines and to 0.002 A in the case of two. Summing up the two investigations, we conclude that there are no errors n relative wave-length of the 53 secondary standards exceeding 0.002 A.

2. The secondary standards belong to groups a, b, c 4, c 5, and *sub-d*, and the quality of the lines of the several groups is in the order given for the groups.

3. It would be a gain if all the lines of group *sub-d* could be replaced by lines of better quality. This is easily feasible for the region λ 5192–λ 5324, where excellent lines of group a are well distributed.

4. From a comparison of the measured intervals between close pairs of lines it appears that the wave-lengths of the components cannot be determined with sufficient accuracy to fit them for standards.

5. In the case of reversed lines, measurements made upon the sharp reversals are the more reliable.

6. The change in relative wave-length between Pasadena and Mount Wilson is less easy to substantiate in the blue-green region, but it appears distinctly in the means.

7. The large discrepancies between observers in the case of lines of groups d and e appear to us attributable to the marked unsymmetrical broadening of these lines near the negative pole of the arc.

8. To eliminate the effect of this polar dissymmetry we recommend that the slit of the spectrograph be placed at right angles to the axis of the arc at the middle point of the enlarged image.

9. The consistency of the various series of our investigation we attribute to the two constant factors, the complete analyzing action of the plane grating and the uniform arc conditions.

MOUNT WILSON SOLAR OBSERVATORY
June 20, 1913
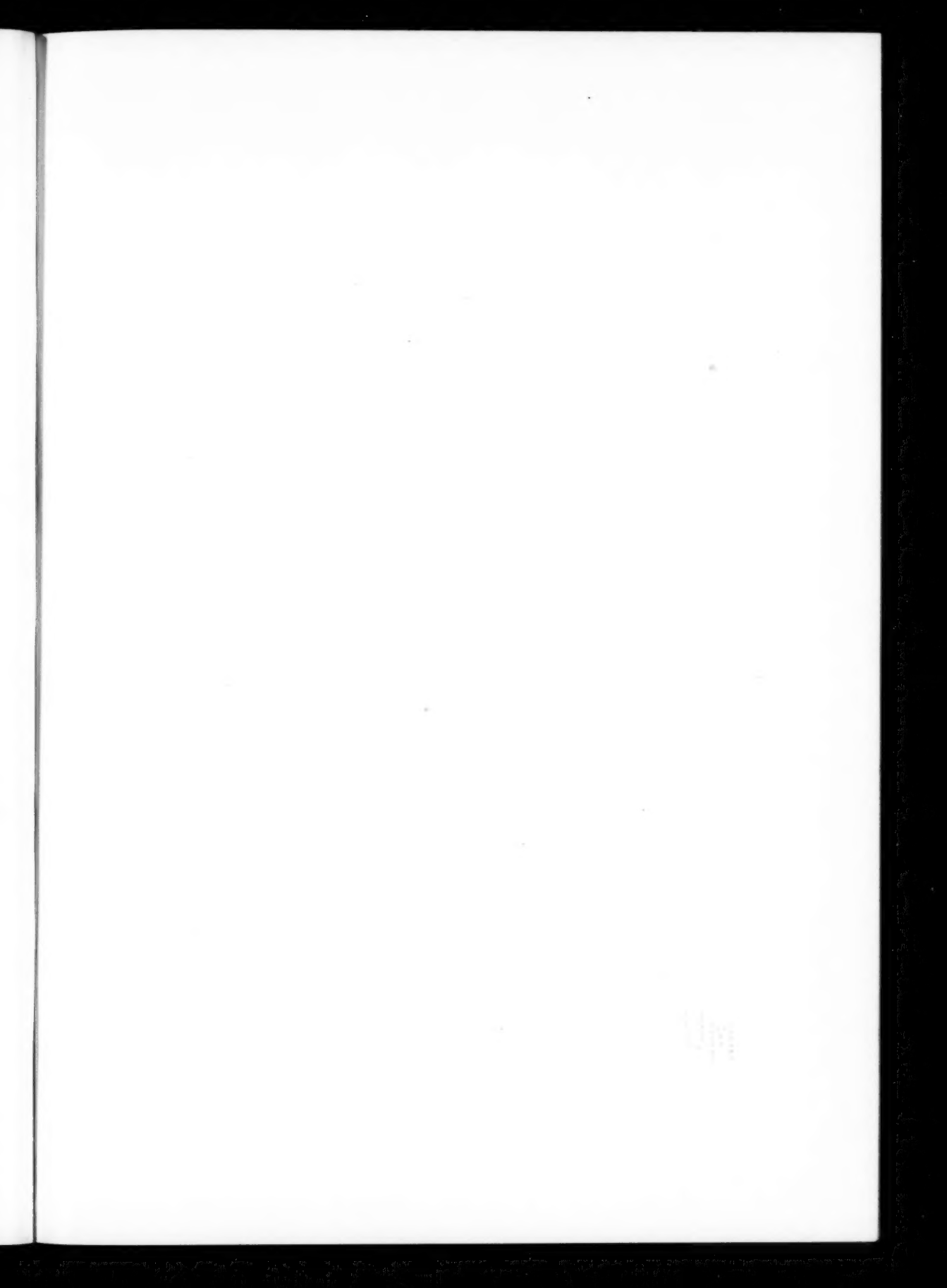
PLATE I
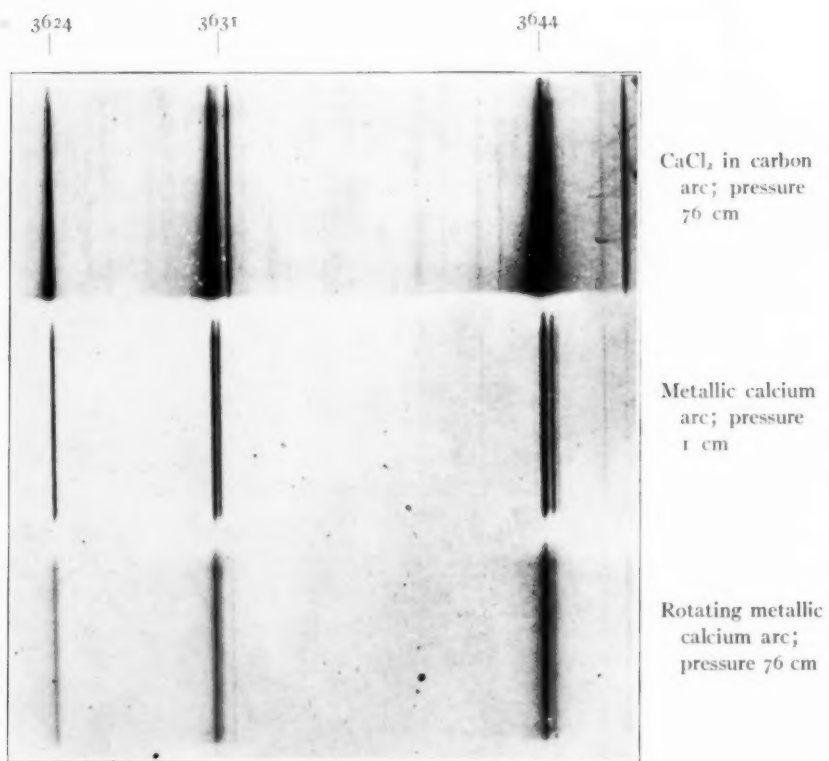
WAVE-LENGTHS IN THE SPECTRUM OF THE CALCIUM ARC *IN VACUO*

By HENRY CREW and GEORGE V. McCAULEY

The establishment of the angstrom in terms of the meter by Michelson and Benoit, followed by the determination of secondary interferometer values by Fabry and Buisson, Eversheim, Pfund, and Burns, marks a new era in quantitative spectroscopy. Kayser's interpolation of tertiary iron standards between these interferometer values completes, in a certain sense, a third step and leads to what is apparently the next problem in spectroscopy; namely, the measurement of all the lines in each of the elements in terms of the international unit of wave-length. A considerable number of such measurements has already been made. A list of these, brought up to date, is given by Kayser;[1] besides these there are certain elements for which the work has been done by interferometer methods, with greater accuracy, but with less completeness. As new spectroscopic sources come into use and new series formulas are suggested, the importance of these determinations increases. It was with this idea in mind that the following measures upon the calcium arc spectrum were undertaken. About the time of their completion appeared Holtz's[2] measures upon the same element. But since his arc was used at atmospheric pressure, and since this pressure is sufficient to widen asymetrically many calcium lines and completely to fuse together certain doublets in one of the important series—as illustrated by Plate I, it has seemed advisable to publish our values which were obtained from an arc working at the lowest practical pressure—about one centimeter of mercury— and giving sharp lines, as already demonstrated by Barnes[3] and Saunders.[4]

[1] *Zeitschrift für wissenschaftliche Photographie*, **12**, 306, 1913.

[2] *Ibid.*, **12**, 101–122, 1913.

[3] *Astrophysical Journal*, **30**, 14, 1909.

[4] *Ibid.*, **32**, 154, 1910.

SOURCE

The electrodes were of metallic calcium, prepared by electrolysis; their cross-section was about 0.35 cm^2; the average separation between electrodes—length of arc—was about 0.5 cm.

The vacuum chamber containing the arc, and shown in Fig. 1, is similar to those used by Barnes[1] and Saunders.[2] The two electrodes pass through asbestos packing at opposite ends of a cylindrical brass chamber, thus enabling the operator to start the arc readily and to adjust for collimation. This chamber consists of two parts which unite, by means of flanges and clamps, at
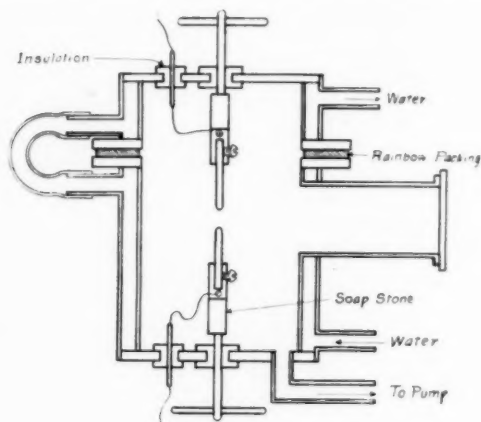
Fig. 1.—Vacuum Arc Chamber

a horizontal plane passing just above the center. In order to secure an air-tight joint a gasket of "rainbow packing" is clamped between these flanges. A side tube permits the light of the arc to pass to the image-lens. A second side tube, at right angles to the first, enables the operator to examine the arc conveniently while exposures are in progress. The chamber is cooled by a small stream of tap-water flowing, in series, through two cylindrical jackets, one about each half of the chamber.

The pressure in the chamber—approximately one centimeter of mercury—was maintained by a Geryk pump and measured

[1] *Astrophysical Journal,* **27**, 153, 1908.   [2] *Loc. cit.*

with a closed-end mercury manometer. As a spectroscopic source, free from noise, annoying vapors, and unsteadiness, the calcium arc under the above conditions leaves little to be desired.

For a comparison spectrum, the middle of a Pfund[1] iron arc was employed.

SPECTROGRAPH

Our photographs were obtained from a ten-foot Rowland concave grating, 9 cm wide, ruled with a total of 50,000 lines, and mounted on a steel triangle in the Rowland method. This triangle was carried upon three brick piers, completely detached from the building and resting directly upon the ground; these three piers were further isolated by being placed in a room inclosed only by interior walls; the steel triangle was isolated from the piers by blocks of soft rubber. From λ 6200 to λ 2800 the second-order spectrum was used; outside this region the first-order was employed.

The photographic plates employed were Cramer "Crown" and Cramer "Spectrum." They were uniformly exposed in the following manner. Immediately in front of the sensitive plate was placed a brass screen with a single horizontal slot 3 mm wide. Through this slot the same part of the plate was exposed in succession to the iron arc during *half* of its proper exposure, then to the calcium arc during the *whole* of its proper exposure, then to the iron arc during the remaining *half* of its proper exposure. Throughout these three exposures nothing about the spectrograph was touched or changed except the source of light. The spectrum strip thus obtained was the one used for measurement. On the upper side of it and in immediate juxtaposition with it was photographed the spectrum of iron alone; on the lower side, in like manner, was photographed the spectrum of calcium alone. These two outside spectra were exposed a little longer than the corresponding spectra on the middle strip and were used merely for the visual identification of *Ca* and *Fe* lines.

During all the five exposures above described the images of the arc covered the same portion of the slit. It is to be noted

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[1] *Astrophysical Journal*, **27**, 296, 1908.

also that the passage from the top to the bottom spectrum on each plate was made by moving the plate and not the metal screen; therefore in all three strips one used the same central portion of the spectral band produced by the grating. In any given region of the spectrum three times of exposure were used for the calcium, viz., one just long enough to yield the strong lines as sharp as possible, one long enough to make lines of medium intensity as sharp as possible, and one to give the weak lines as great density as possible.

MEASUREMENT OF PLATES

All the second-order and some of the first-order plates were measured on a Société Genévoise dividing engine. The screw of this engine is 35 cm long and has a pitch of one millimeter. By means of an extra gear wheel, a Veeder cyclometer, and a correcting bar—a combination due to our colleague, Professor Tatnall—this engine enables one to read directly to hundredths of an angstrom; and since a division of the head, indicating one-hundredth of an angstrom, is approximately 2 mm long, one can easily estimate to thousandths of an angstrom.

The wave-lengths from the first-order plates were also obtained by means of a very superior measuring engine made by Mr. F. Küng, the university mechanician.   The screw of this engine was cut with a pitch which differed by only one part in ten thousand from a length equal to 10 angstroms on the first-order plates.   On the head of this screw one-hundredth of an angstrom corresponds to 0.35 mm in length.

The wave-lengths were determined as follows: The reading microscope and cyclometer were so adjusted that the reading of the engine for a standard iron line near one end of the plate was identical with its wave-length. The successive positions of all the lines appearing in the calcium spectrum and of a sufficient number of standard iron lines were read.   This process was repeated twice.   The mean of these three sets, considered as a single determination, was then corrected as follows: A curve was plotted in which wave-lengths were used as abscissae, while the differences between the wave-lengths of the iron standards and our engine readings for these same lines were employed as ordinates. A

TABLE OF WAVE-LENGTHS

(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
2103.239[2]	2	.1	+0.14	P_1	3678.240	2	....	.....	t
12.763[2]	2	.7	+0.06	P_1	3706.022	9	.028	−0.006	P_2
13.19 [3]	1*	....	.....	P_1	36.903	9	.905	−0.002	P_2
50.78 [3]	1	....	.....	SL_1	48.374[1]	1	....	.....	t
97.791[2]	3	.8	−0.01	P_2	50.349	1	....	.....	t
2200.76 [3]	1	.5	+0.26	SL_1	53.367[1]	1	....	.....	t
08.606[2]	3	.7	−0.09	P_2	3870.506	2	....	.....	t
75.493[3]	1	.5	−0.01	SL_1	72.552	3	....	.....	t
2398.582	2	.587	−0.005	SL_1	75.807	4	....	.....	t
2994.953	4	.961	−0.008	T	89.141	1	....	.....	SL_3
97.309	4	.311	−0.002	p	3933.664	10R	.674	−0.010	PH
99.651[1]	4	.653	−0.002	T	46.05	0	....	.....	SL_2
3000.865[1]	4	.878	−0.013	..	48.899	3	.914	−0.015	T_2
06.864	5	.859	+0.005	p	52.84	000	....	.....	..
09.212	4	.210	+0.002	T	57.054	5	.063	−0.009	T_2
3102.36	0	....	.....:	T_2	68.465	10R	.479	−0.014	PH
07.388	1	....	.....	T_2	72.578	1*	....	.....	SL_3
17.656	1	....	.....	T_2	73.716	4	.718	−0.002	T_2
36.003	2	5.868	+0.135	T_1	4058.912	0	....	.....	SL_2
40.782	2	.720	+0.062	T_1	92.649	2	.690	−0.041	t
41.164	0*	....	.....	T_1	94.944	3	.983	−0.039	t
50.747	4	.714	+0.030	T_1	98.552	4	.575	−0.023	t
51.280	1*	....	.....	T_1	4108.554	1	....	.....	SL_3
58.877	10	.866	+0.011	P_1	4226.731	10R	.728	+0.003	..
64.618	1	....	.....	T_2	40.455	2	.437	+0.018	SL_2
69.854	1	....	.....	T_2	83.008	9	.006	+0.002	p
79.340	10	.332	+0.008	P_1	89.363	9	.362	+0.001	T
80.521	2*	....	.....	T_1	98.989	8	.988	+0.001	T
81.283	6	.274	+0.009	P_1	4302.525	9	.528	+0.001	p
3209.930	3	.892	+0.038	T_1	07.738[1]	7	.744	−0.006	..
15.145	3	.126	+0.019	T_1	18.648	9	.645	+0.003	T
15.334	1*	....	.....	T_1	55.099	5	.14	−0.041	SL_3
25.883[1]	5	.862	+0.021	T_1	4425.428	9	.449	−0.021	T_1
26.129[1]	1*	....	.....	T_1	34.948	9	.963	−0.015	T_1
69.090	1	.139	−0.049	T_2	35.673	8	.682	−0.009	T_1
74.661	2	.703	−0.034	T_2	54.765	9	.781	−0.016	T_1
86.060	3	.100	−0.040	T_1	55.875	5	.893	−0.018	T_1
3344.508	5	.491	+0.017	T_1	56.612	3	.623	−0.011	T_1
50.198	5	.188	+0.010	T_1	4507.854	o*wh*	....	.....	..
50.361	2*	....	.....	T_1	09.446	0	....	.....	..
61.918	6	.904	+0.014	T_1	12.281	1	....	.....	..
62.131	2*	....	.....	T_1	26.944	4	.952	−0.008	SL_3
62.28	0*	....	.....	T_1	78.570	4	.588	−0.018	t
3468.484	2	.489	−0.005	T_2	81.414	5	.445	−0.031	t
74.774	3	.779	−0.005	T_2	85.868	6	.908	.....	t
87.611	5	.613	−0.002	T_2	85.923	2	....	.....	t
3624.107	6	.106	+0.001	T_1	4685.264	2	.273	−0.009	..
30.749	6	.739	+0.010	T_1	4807.54	oo*wh*	....	.....	..
30.973	2	.958	+0.015	T_1	23.08	oo*wh*	....	.....	..
44.400	7	.403	−0.003	T_1	33.94	oo*wh*	....	.....	..
44.760	3	.757	+0.003	T_1	47.292	2	....	.....	..
44.990	0*	....	.....	T_1	78.132[2]	5	.168	−0.036	SL_3
73.448	1	....	.....	t	5001.489	1̵	....	.....	vp
75.307	2	....	.....	t	19.981	2	....	.....	vp

TABLE OF WAVE-LENGTHS—*Continued*

(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
5021.141...	0	....	......	vp	6161.309...	2	.321	—0.012	..
41.613¹) .	3	.634	—0.021	SL₂	62.177...	9	.196	—0.019	T₂
5188.846...	3	.854	—0.008	..	63.749...	2	.797	—0.048	..
5260.375...	1	.387	—0.012	..	66.443...	2	.486	—0.043	..
61.701...	3	.698	+0.003	..	69.034...	2	.081	—0.047	..
62.238...	3	.239	—0.001	..	69.576...	3	.605	—0.029	..
64.237...	3	.235	+0.002	..	6417.71....	00	....	......	..
65.559..	5	.551	+0.008	..	39.086...	9	.061	+0.025	..
70.272¹) .	5	.265	+0.007	..	49.811..	7	.794	+0.017	p
5349.470...	5	.467	+0.003	..	55.606...	3	.560	+0.046	p
5512.978...	4	.935	+0.043	..	62.576...	9	.550	+0.026	..
81.973...	6	.956	+0.017	..	71.659...	5	.644	+0.015	..
88.746...	9	.743	+0.003	..	93.789...	8	.762	+0.027	p
90.109...	5	.099	+0.010	..	99.648...	4	.624	+0.024	p
94.464...	8	.447	+0.017	..	6508.742...	1	....	......	..
98.484...	8	.467	+0.017	..	72.783...	3	.71	+0.073	..
5601.283...	5	.264	+0.019	..	6707.866...	1	.81	+0.056	..
5602.829¹) .	5	.844	—0.015	..	17.688...	5	.70	—0.012	..
5857.476...	8	.494	—0.018	..	7148.123...	3	....	......	p
67.578...	1	.61	—0.032	..	7202.161...	2	....	......	p
6102.716...	8	.736	—0.020	T₂	7326.099...	1	....	......	..
22.216...	8	.232	—0.016	T₂					

¹) Too near iron line for accurate measurement; measured with reference to other calcium lines.
²) Photographed in the third-order and measured with reference to second-order iron lines.
³) Measured by interpolation between those calcium lines designated by "2)."

smooth curve drawn through the points thus located gave us graphically the correction needed to reduce the readings of each line to the scale of the iron standards; i.e., to terms of the international unit.

RESULTS

In this manner were obtained the wave-lengths of the calcium arc *in vacuo* given in the first column of the above table; in column 2 follow the intensities on the customary scale running from 1 to 10; in column 3 are the values obtained by Holtz for the arc at atmospheric pressure, using $CaCO_3$ in cored carbons; in column 4 are placed the divergences between our values and those of Holtz; in column 5 is indicated in Saunders'[1] notation the series to which the line belongs.

Abbreviations used in the table are as follows: *R* denotes a reversed line; *wh*, one which is wide and hazy; an asterisk indicates a line hitherto unobserved.

[1] *Astrophysical Journal*, **32**, 178, 1910.

DISCUSSION

The lines measured include all those found by Kayser and Runge,[1] with the exception of $\lambda\lambda$ 4624.71, 3166.95, and 3101.87. These last two are to be identified probably with our lines at 3164.62 and 3102.38, which satisfy more nearly the series formula proposed by Kayser and Runge. Of the new lines discovered by Saunders in the region studied, the following eighteen were not observed by us:

1837.1	2083.2	2216.7	2335.0	5342.4	6784.35
1839.8	2097.8	2249.8	2354.7	6261.7	6789.6
2072.8	2118.99	2287.9	2373.3	6395.4	6798.9

For the sake of completeness, reference is also made to the lines of calcium measured by Paschen[2] and Hermann[3] in the infra-red, and to those measured by Lyman[4] in the ultra-violet.

Of the eleven new lines observed by us, all belong to the first subordinate series of main triplets, except $\lambda\lambda$ 3180.52, 3972.58, and 2113.19. The line at 3180.52 probably belongs to the second subordinate series of main triplets, replacing λ 3181.28 employed by Kayser and Runge,[5] and fills the place left vacant in Saunders' classification,[6] which makes λ 3181.28, from its close relationship with the pair $\lambda\lambda$ 3158.88 and 3179.34, a satellite of λ 3179.34. The line at 3972.58 appears to be the missing member of Saunders'[7] series SL_3, while λ 2113.91 appears to be a satellite of the longer line in the pair $\lambda\lambda$ 2103.24 and 2112.76 of the first subordinate series of pairs. It is therefore the line analogous to λ 3181.28. The new lines in the structure of the first subordinate series of main triplets are analogous to $\lambda\lambda$ 4435.67, 4455.88, and 4456.62, as may be readily seen from the table. λ 3362.28 may be one of the spurious lines accompanying λ 3361.92 and due to this grating, which has the peculiarity of giving several sharp companions to each strong line, all within the first ghost distance. However, on the two plates from which λ 3362.28 was measured, these

[1] *Annalen der Physik,* **43**, 391, 1891.

[2] *Ibid.,* **20**, 651, 1909.   [3] *Loc. cit.*

[3] *Ibid.,* **16**, 684, 1905.   [6] *Astrophysical Journal,* **32**, 169, 1910.

[4] *Astrophysical Journal,* **35**, 341, 1912.   [7] *Ibid.,* **32**, 157, 1910.

spurious lines were not prominent and seemed to be clearly separated from the line in question. We therefore give this wavelength as a "candidate" for the position in the calcium triplet at this point analogous to each of the positions occupied by λ 4456.61 and λ 3644.99 in the longer triplets of this same series.

The divergences between our measures and those of Holtz are seldom greater than one-hundredth of an angstrom; the principal exceptions are those lines characterized by Holtz as being hazy or widened either to the red or to the violet. A few noticeable cases in which the foregoing statement does not apply are worthy of mention. The divergence in the measures of λ 3286.06 is explained by ascribing to it the same physical character as to the other two lines, $\lambda\lambda$ 3274.66 and 3269.09, of this triplet. A similar statement holds in the case of the "narrow triplet," λ 4578.57, λ 4581.41, and λ 4585.87, which should have the same appearance— a widening to the red—as the next triplet, λ 4092.65, λ 4094.94, and λ 4098.55, in the same series. The large discrepancy in λ 5512.98 and the systematic differences in the measures of all lines longer than λ 5800, we are unable to explain.

The average divergence between our own measures, made from different plates, and by different observers, was only six- or seven-thousandths of an angstrom unit. This seemed to warrant the use of the third decimal in the final values except for a few very weak and difficult lines which have been entered in the table with two decimal figures only.

The identity of λ 6707.87 has not been satisfactorily established. Adams[1] thinks that it may be an impurity, possibly lithium; while Holtz[2] concludes positively that it is due to calcium, since other lithium lines of equal intensity do not appear on any of his photographs. We have included the line in our list without undertaking at this time to ascertain its origin.

PRINCIPAL IMPURITIES

In the calcium were found as impurities: magnesium, strontium, manganese, silicon, sodium, iron, and aluminium.

[1] *Astrophysical Journal*, **30**, 92, 1909.

[2] *Loc. cit.*

Reliable measures were made on the following lines:

Mg		*Sr*	*Mn*	*Na*
2795.540	3838.296	4077.723	4030.754	5889.963
2852.132	4481.104	4215.525	4033.064	5895.929
3829.361	5172.642	4607.336	4034.484	
3832.308	5183.598			

UNIDENTIFIED LINES

The following twenty-six lines which appeared repeatedly on the photographs are unidentified: it is, of course, not impossible that some of them belong to calcium.

2721.646	3045.751	3099.341	3795.618	4506.624
3018.551	3055.321	3461.896	4109.825	4507.417
3024.927	3071.575	3108.577	4110.330	4547.878
3034.524	3076.988	3594.083	4137.922	6259.753
3041.046	3080.819	3683.714	4496.158	6456.907
		3694.108		

COMPARISON OF ARC AND SPARK SPECTRA

A comparative study of the spark at reduced pressure was made from a series of photographs of the arc and spark taken side by side. The same electrodes and same vacuum chamber were used for the spark as for the arc. Step-up transformers were employed to furnish the potential needed for the spark. One of these has a closed magnetic circuit and transforms from 100 to 5000 volts; the other has an open magnetic circuit and transforms from 100 to 2500 volts; but the latter furnishes much greater current in the spark circuit than can be obtained with the former. The spectrum of the spark under a pressure of one centimeter of mercury was found to be relatively weak in calcium and strong in cyanogen and nitrogen bands. The heads of these bands appear stronger in the spark, while the "structure" lines are stronger in the arc. At a pressure of 48 cm the photographs are entirely free from bands of cyanogen and nitrogen, and more calcium lines make their appearance.

The only lines appearing in the spark and not in the arc were identified either as air lines or "spark" lines of impurities. Even with exposures that bring out the faint lines $\lambda\lambda$ 3644.76, 3644.99,

3350.36, and 3344.51 of the first subordinate series of main triplets, no trace is found of the lines listed by Eder and Valenta[1] as characteristic of the spark.   Nor did λ 2373.27 and λ 4132.7 of Exner and Haschek[2] appear.

BAND SPECTRA

In addition to the bands already studied by Olmstead[3] and Eagle,[4] the arc spectrum of calcium, at reduced pressure, shows several bands which make their appearance only during the first half-hour of exposure with fresh calcium electrodes.   These lie in the region λ 6700–λ 7000 and at λ 3500, and present a structure of fairly sharp lines with no continuous background.   The origin of these bands we have left undetermined.

PHYSICAL LABORATORY
NORTHWESTERN UNIVERSITY
     September 27, 1913

[1] *Denkschriften der Wiener Akademie*, **67**, 1898.

[2] *Die Spectren der Elemente*, Wien, 1911.

[3] *Astrophysical Journal*, **30**, 231, 1909.

[4] *Ibid.*, **27**, 66, 1908.

MEASURES OF VARIABLE RADIAL VELOCITIES OF STARS

By OLIVER J. LEE

In the course of measuring Bruce spectrograms the ten stars given in the first list below were found to be binaries. In the fourth column, "Observer," A=Adams; Ar=Arbogast; B=Barrett; F=Frost; L=Lee; M=S. A. Mitchell. F. R. Sullivan has assisted in observing, as usual.

89f Piscium ($a = 1^h 13^m$; $\delta = +3°5'$; Mag. $= 5.3$)

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 2108..........	1909	Aug. 20	$21^h 37^m$	L	6	$+27$	g.
2120..........		Aug. 27	20 50	L	9	$+16$	g.
2494..........	1910	Aug. 25	20 6	L	6	$+18$	v.w.
2569..........		Nov. 28	14 40	F	5	-12	w.

g. = good; v. = very; w. = weak.

The spectrum is classified as of type A2. It has numerous metallic lines which are diffuse and on all the plates give the effect of unresolved components of unequal intensity. On plate No. 2120 the edges of the lines were sharp and supposed components were measured, giving -30 km and $+73$ km from 8 and 6 lines respectively. *Mg* λ 4481 alone shows a range of velocities of 45 km when referred to the sun.

73 ξ^2 Ceti ($a = 2^h 23^m$; $\delta = +8°1'$; Mag. $= 4.3$)

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Velocity λ 4481	Quality
						km	km	
IB 2123.....	1909	Aug. 30	$19^h 43^m$	B	9	$+ 4$	$+ 4$	v.g.
2132.....		Sept. 10	19 59	L	7	$+ 6$	$+ 9$	g.
2547.....	1910	Oct. 28	17 31	B, Ar	4	$+20$	$+24$	v.g.
2559.....		Nov. 7	15 10	F, Ar	5	$+13$	$+13$	v.g.

The spectrum is of class A. The *Mg* line λ 4481 is by far the best on our plates and velocities derived from it were reduced to the

sun and tabulated as above after duplicate measures on it had been made.

125 Tauri ($\alpha = 5^h 34^m$; $\delta = +25°50'$; Mag. $= 5.0$)

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 2887.........	1911 Dec. 3	$17^h 57^m$	L	7	$+13$	v.g.
3194.........	1912 Dec. 10	17 36	M	6	$+63$	g.
3223.........	Dec. 30	15 45	B	3	$+1$	v.g.

This star has a spectrum of type B3. The lines are simple.

40 Aurigae ($\alpha = 6^h 0^m$; $\delta = +38°30'$; Mag. $= 5.3$)

PLATE	DATE	G.M.T.	TAKEN BY	STRONGER COMP.		WEAKER COMP.		QUAL-ITY
				Velocity	No. of Lines	Velocity	No. of Lines	
				km		km		
IB 3512..	1913 Oct. 2	$22^h 11^m$	L	-58.5	12	$+130.8$	9	g.
3517..	3	21 24	L	-35.6	16	$+92.9$	10	g.
3519..	6	21 31	B	$+22.8$	7			w.

This star, of spectral type A, has numerous good metallic lines. No. 3519 shows the two components superposed. The orbits will be discussed when sufficient data have been obtained.

24 Canum Venaticorum ($\alpha = 13^h 30^m$; $\delta = +49°32'$; Mag. $= 4.6$)

PLATE	DATE	G.M.T.	TAKEN BY	CENTER		VIOLET COMP.		RED COMP.		MEASURED BY	QUALITY
				No. of Lines	Velo-city	No. of Lines	Velo-city	No. of Lines	Velo-city		
					km		km		km		
IB 1983	1909 Feb. 5	$21^h 24^m$	L	6	-32	2	$+8$	5	$+34$	Ar	g.
				4	-4	5	-38	4	$+58$	L	
2337	1910 May 20	16 56	B	7	-26	4	-51	4	$+40$	Ar	g.
				10	-22	3	-60	3	$+26$	L	
2662	1911 Jan. 23	20 57	B	6	$+1$	1	-45	1	$+72$	Ar	w.
				7	$+14$	4	-41	4	$+65$	L	
2668	Jan. 27	22 4	Ar	5	$+18$	11	-33	6	$+64$	L	g.
2694	Mar. 3	19 30	Ar	14	-8	1	-58	1	$+20$	Ar	g.

This spectrum, of type A3, has numerous lines which are diffuse and hard to analyze when complicated by doubling. The com-

ponents are of nearly equal brightness. On No. 2668 Arbogast measured the middle points of 4 lines, the mean of which reduced to the sun is -23 km. On No. 2694 my velocity for the center is -16 km from 8 lines. Our measures were made independently and the binary nature of the star was first noted by Arbogast.

33 Boötis ($\alpha = 14^h 35^m$; $\delta = +44°50'$; Mag. $= 5.4$)

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 3282........	1913	Feb. 5	$22^h 31^m$	L	5	$+15$	g.
3331........		Mar. 31	18 42	L, M	4	-6	v.g.
3348........		Apr. 16	17 26	B	4	-21	v.g.

This is an A-type spectrum. The metallic lines are faint but measurable. .

27 β Librae ($\alpha = 15^h 12^m$; $\delta = -9°1'$; Mag. $= 2.7$)

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 337........	1904	May 10	$14^h 58^m$	F, B	3	-18	v.g.
2308........	1910	Mar. 14	21 2	B	4	-29	v.g.
2334........		May 13	19 16	L	4	-53	g.
2340........		May 20	19 51	L	3	-65	v.g.
2698........	1911	Mar. 3	23 23	Ar	4	-12	v.g.

This spectrum is of type B8 and the hydrogen lines alone are measurable. Low dispersion and fine-grained plates should be used in further investigation of it.

B.D. $25°4165$ ($\alpha = 20^h 11^m$; $\delta = +25°17'$; Mag. $= 4.8$)

Plate	Date		G.M.T.	Taken by	Center		Violet Comp.		Red Comp.		Quality
					No. of Lines	Velocity	No. of Lines	Velocity	No. of Lines	Velocity	
						km		km		km	
IB 47	1903	June 13	$19^h 22^m$	A	6	-14	.....		.....		v. strong
59		July 24	20 14	F	3	-23	3	-85	3	$+30$	v.g.
69		Sept. 5	15 22	A	3	-15	1	-80	1	$+40$	v.g.
78		Sept. 18	13 31	A	2	$+20$	2	-40	2	$+88$	v.g.
372	1904	June 18	20 19	B	5	-11	6	-58	5	$+55$	g.

This star has a spectrum of type B3. The lines are diffuse and hard to analyze. The line Mg λ 4481 is widely double on No. 372 and is certainly single on No. 59. Without doubt this star is a binary showing two spectra. The spectrum shows striking changes, possibly due to causes other than those of velocity.

33 ι Aquarii $(\alpha = 22^h 1^m;$ $\delta = -14° 21';$ Mag. $= 4.4)$

Plate	Date		G.M.T.	Taken by	Stronger Component	Weaker Component	Quality
					km	km	
IB 1755.........	1908	Sept. 25	14^{h}36^m	B	−68	+ 46	g.
1762.........		Sept. 29	14 47	L	+ 9	−130	g.
1766.........		Oct. 2	16 35	B	−87	+ 48	w.
1780.........		Oct. 9	13 10	L	−97	+ 97	g.
1789.........		Oct. 12	12 27	L	−66	+103	g.

The measures given above are on the Mg line λ 4481 alone, this being the only interpretable line in the spectrum. The character of the line changes greatly. The period may be short. The spectrum is of type B8.

18 λ Piscium $(\alpha = 23^h 37^m;$ $\delta = +1° 14';$ Mag. $= 4.6)$

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Measured by	Quality
						km		
IIB 3....	1903	Sept. 19	16^{h}45^m	F	5	+11	F	p.
					11	+ 5	L	
IB 91....		Sept. 25	16 36	F	9	+19	F	g.
					10	+23	L	
199....		Dec. 1	12 29	F, A	7	+ 6	F	v.g.
					10	+ 8	L	
217....		Dec. 25	12 19	A, F	7	+10	F	g.
					13	+ 9	L	
585....	1905	Sept. 18	17 11	F	6	+36	L	w.
1238.....	1907	Nov. 22	12 51	B, L	10	+12	L	v.g.
2764.....	1911	July 14	20 55	B	11	+20	L	v.g.

This spectrum is of type A5. It has numerous good lines which seem to be overlapping components. An attempt was made to measure them, but similar lines did not show the proper consistency and the results are therefore not given. The dispersion of two and three prisms can be used to advantage on this star. Duplicate measures on No. 585 gave +34 and +38 km from 4 and 6 lines respectively.

The following eighteen stars have been previously announced here or elsewhere as spectroscopic binaries. The plates here measured were obtained for the most part in the course of the regular program with the Bruce spectrograph, generally prior to the announcement of the variable velocity of the stars by other observers. The director has removed all of these stars from the observing program for the present.

15 κ Cassiopeiae ($\alpha = 0^h 27^m$; $\delta = +62°23'$; Mag. = 4.2)

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 1718........	1908	Sept. 7	$16^h 25^m$	B	9	− 3	g.
1727........		Sept. 8	15 10	L	10	− 8	g.
1772........		Oct. 5	15 7	F, L	8	+ 2	v.g.
2475........	1910	Aug. 8	18 38	L, B	5	−16	w.
2482........		Aug. 12	19 46	L	7	−10	g.

The variable velocity of this star was announced in the *Lick Observatory Bulletin*, **6**, 141, 1911.

29 π Andromedae ($\alpha = 0^h 32^m$; $\delta = +33°10'$; Mag. = 4.4)

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 1177........	1907	Sept. 23	$19^h 6^m$	L	7	+67	v.g.
1185........		Sept. 24	18 29	F, L	6	+68	v.g.
1197........		Oct. 11	17 28	B	5	+26	g.
1230........		Oct. 22	15 41	F	7	+14	v.g.

This binary was announced by Frost and Adams in this *Journal*, **18**, 384, 1903, from their measurement of three earlier spectrograms.

46 ω Cassiopeiae ($\alpha = 1^h 48^m$; $\delta = +68°12'$; Mag. = 5.0)

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 2142........	1909	Sept. 20	$18^h 45^m$	L, B	3	−35	w.
2799........	1911	Aug. 18	20 12	B	4	−28	g.
2862........		Nov. 20	12 9	L	6	− 7	w.
2863........		Nov. 25	13 49	B	5	− 1	g.

This spectrum is of type B8 and the hydrogen and helium lines are strong and fairly sharp. The variable velocity was announced by Adams in *Publications of the Astronomical Society of the Pacific*, **24**, 129, 1912.

<p align="center">4 g Persei (a = 1^h56^m; δ = +54°0′; Mag. = 5.0)</p>

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 1171.........	1907	Sept. 21	20^h44^m	B	4	− 6	v.g.
1187.........		Sept. 24	20 47	L	3	−17	g.
1283.........		Dec. 11	11 47	L, B	3	0	g.
1288.........		Dec. 16	12 54	B	5	+ 1	g.
1328.........	1908	Jan. 14	16 50	B	4	+ 2	g.

The variable radial velocity of this star was announced by Frost and Adams in this *Journal*, **19**, 152, 1904, where will be found their measures of four earlier plates.

<p align="center">33 τ⁸ Eridani (a = 3^h49^m; δ = −24°54′; Mag. = 4.8)</p>

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 639.........	1905	Dec. 15	15^h37^m	F	7	+27	g.
649.........		Dec. 25	15 34	F	5	+22	g.
657.........	1906	Jan. 26	14 0	F	6	+29	v.g.
885.........		Oct. 19	19 20	B	7	+ 8	v.g.
903.........		Nov. 1	18 57	F	3 •	+ 8	g.
1320.........	1908	Jan. 13	15 42	L	4	+ 3	g.

The binary character of this star was announced by Frost in this *Journal*, **25**, 64, 1907, from preliminary measures on the first five plates given above. My measures of them and of No. 1320 are to be considered as definitive. Mr. Ichinohe assisted in securing four of these plates.

<p align="center">45 ε Persei (a = 3^h51^m; δ = +39°43′; Mag. = 3.0)</p>

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 1176.....	1907	Sept. 23	18^h25^m	L	3	−22	v.g.
1188.....		Sept. 24	21 31	L	6	+13	v.g.
1193.....		Oct. 7	22 33	L	5	Max. −71	v.g.
					4	Min. +67	

Measures of five earlier plates of ϵ *Persei* were published in this *Journal*, **19**, 152, 1904, by Frost and Adams. These observers commented upon the complexity and changes in the lines but did not measure components.

94 τ Tauri ($\alpha = 4^h 36^m$; $\delta = 22°46'$; Mag.$= 4.3$)

Plate	Date			G.M.T.	Taken by	No. of Lines	Velocity	Quality
							km	
IB 443.........	1904	Nov.	8	$17^h 42^m$	F	6	+ 6	v.g.
466.........		Dec.	30	14 35	F	8	+39	v.g.
588.........	1905	Sept.	18	19 58	B	2	−17	w.
1286.........	1907	Dec.	11	14 50	L	6	+51	g.

This helium star was announced as a binary by Frost and Adams in this *Journal*, **17**, 246, 1903, from the measures of three early high-dispersion plates taken with a short camera. The orbit has been published by Parker in *Journal of the Royal Astronomical Society of Canada*.

20 τ Orionis ($\alpha = 5^h 13^m$; $\delta = -6°57'$; Mag.$= 3.7$)

Plate	Date			G.M.T.	Taken by	No. of Lines	Velocity	Quality
							km	
IB 931.........	1906	Dec.	17	$17^h 3^m$	F	4	+29	g.
968.........	1907	Feb.	2	15 18	Fox	8	+26	v.g.
1399.........	1908	Feb.	2	16 29	Jordan	3	+25	w.

The binary nature of this helium star was announced in this *Journal*, **25**, 64, 1907, by Frost.

30 τ Canis majoris ($\alpha = 7^h 15^m$; $\delta = -24°47'$; Mag.$= 4.4$)

Plate	Date			G.M.T.	Taken by	No. of Lines	Velocity	Quality
							km	
IB 712.........	1906	Mar.	23	$14^h 19^m$	F	2	+78	v.w.
714.........		Mar.	30	14 4	F	5	+48	g.
722.........		Apr.	2	13 30	F	7	+38	g.
974.........	1907	Feb.	7	15 48	F	2	+ 0	v.w.
1991.........	1909	Mar.	1	15 52	F	8	+92	g.

The variable velocity of this star was announced by Frost in a footnote on p. 265 of the *Astrophysical Journal*, **23**, 1906. Measures of four Lick plates are given in *Lick Observatory Bulletin*, **6**, 145, 1911.

8π *Virginis* ($\alpha = 11^{h}56^{m}$; $\delta = 7°10'$;   Mag. $= 4.6$)

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 733.........	1906	Apr. 20	$15^{h}\ 4^{m}$	B	9	-33	g.
755.........		May 11	14 24	B	9	-20	g.
1040.........	1907	Apr. 26	16 14	F	10	$-\ 8$	g.
1979.........	1909	Feb. 1	21 43	B	7	$+\ 6$	g.

This star was announced as a binary by Albrecht in *Lick Observatory Bulletin*, **5**, 175, 1910.

21ι *Boötis* ($\alpha = 14^{h}13^{m}$; $\delta = +51°50'$;   Mag. $= 5.1$)

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 2704.........	1911	May 1	$17^{h}20^{m}$	F, Ar	2	$+\ 3$	Poor
3009.........	1912	Mar. 6	19 49	B	7	$-\ 3$	v.g.
3015.........		Mar. 8	19 30	B	6	-16	v.g.

The spectrum of this star contains numerous rather poor lines. Measures by Jordan are recorded in the *Publications of the Allegheny Observatory*, **2**, 123, 1911. There can be no doubt of the actual variation in radial velocity.

$9 \alpha^{2}$ *Librae* ($\alpha = 14^{h}45^{m}$; $\delta = -15°38'$;   Mag. $= 2.9$)

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
						km	
IB 3010.........	1912	Mar. 6	$21^{h}33^{m}$	B	9	$-\ 4$	v.g.
3016.........		Mar. 8	20 52	B	13	-10	v.g.
3021.........		Mar. 5	19 16	B	7	$+26$	v.g.

The binary character of this star was reported by Slipher in the *Lowell Observatory Bulletin*, **11**, 57, 1904. The hydrogen lines on our Bruce plates are apparently complex, but there is no evidence of the second component in the numerous metallic lines.

14 ι Coronae Borealis $(\alpha = 15^h 57^m; \; \delta = +30°8'; \; \text{Mag.} = 4.9)$

Plate	Date		G.M.T.	Taken by	No. of Lines	Velocity	Quality
IB 2705.........	1911	May 1	$18^h 35^m$	F, Ar	4	km − 3	w.
2723.........		June 9	15 1	B	8	−13	g̣.
3035.........	1912	Mar. 29	20 59	B	8	−20	v.g.

The variable velocity was announced by Baker in *Publications of the Allegheny Observatory*, **1**, 121, 1909.

55 α Ophiuchi $(\alpha = 17^h 30^m; \; \delta = +12°38'; \; \text{Mag.} = 2.1)$

PLATE	DATE		G.M.T.	TAKEN BY	STRONGER COMP.		WEAKER COMP.		QUAL-ITY
					Velocity	Quality	Velocity	Quality	
IB 1165	1907	Sept. 16	$15^h 54^m$	L	km − 16	g.	km +120	w.g.	v.g.
1168		21	14 7	F	+ 4	g.	− 98	g.	v.g.
1174		23	15 5	B	+ 69	....			v.g.
1182		24	13 23	F	− 30	g.	+125	g.	v.g.
1204	Oct.	18	13 10	B	− 79	g.	+ 38	g.	v.g.
1213		20	14 13	F	− 54	g.	+ 86	w.	v.g.
1219		21	13 0	F, L	+127	g. wide	− 77	g. wide	v.g.
1228		22	13 56	F	− 18	Distorted	−140	w. doubtful	v.g.
1546	1908	Mar. 20	23 38	B	+105	Fair	−113	w. wide	v.g.

The spectrum of this star is given as A5. The *Mg* line λ 4481 shows the components best on our plates and the measures given above were made on this line alone. On No. 1168 the components are equally strong and on No. 1174 the line could not be resolved. These observations seem to be roughly satisfied with a period of about 2 days. The binary nature of the star was discovered by Frost and announced in *Astronomische Nachrichten*, **177**, 174, 1908.

102 Herculis ($\alpha = 18^h 4^m$; $\delta = +20°48'$; Mag. $= 4.3$)

Plate	Date			G.M.T.	Taken by	No. of Lines	Velocity	Quality
							km	
IB 123.........	1903	Oct.	17	$13^h 42^m$	A	12	-16.5	v.g.
388.........	1904	July	26	17 12	F	10	-16.3	g.
1636.........	1908	July	20	16 15	B	11	-15.6	v.g.
1645.........		July	24	18 25	B	11	-16.1	v.g.

This star, No. 8382 of Burnham's *General Catalogue*, with a 12th magnitude companion, was one of the twenty helium stars observed by Frost and Adams (*Publication of the Yerkes Observatory*, **2**, 1903). From four plates they obtained a mean velocity of -10.8 km, with a range of 3.9 km. This value is given with revised wave-lengths of the silicon lines as -8.8 km in this *Journal*, **32**, 85, 1910. The binary character of the star was detected by Albrecht (*Lick Observatory Bulletin*, **6**, 147, 1911) with value ranging from -11 to -18 km. The spectrum is of type B2, and numerous lines are well measurable with a dispersion of one prism.

111 Herculis ($\alpha = 18^h 43^m$; $\delta = +18°4'$; Mag. $= 4.4$)

Plate	Date			G.M.T.	Taken by	No. of Lines	Velocity	Quality
							km	
IB 2346.........	1910	May	23	$19^h 50^m$	B	11	-36.2	v.g.
2405.........		June	27	17 35	L	12	-52.1	v.g.
2443.........		July	25	15 38	L	12	-42.2	v.g.
2463.........		Aug.	5	15 24	B	11	-43.1	v.g.

The variable radial velocity of this star was discovered by Wright and it was announced in *Lick Observatory Bulletin*, **5**, 63, 1908.

14 Pegasi ($\alpha = 21^h 45^m$; $\delta = +29°42'$; Mag. $= 5.0$)

Plate	Date			G.M.T.	Taken by	No. of Lines	Velocity	Quality
							km	
IB 2423.........	1910	July	8	$19^h 15^m$	L	7	-30	v.g.
2432.........		July	18	20 3	B	8	-26	v.g.
2467.........		Aug.	5	18 9	B, L	5	-19	g.

The variable velocity of this star was announced in the *Lick Observatory Bulletin*, **6**, 149, 1911.

48 γ Aquarii $(\alpha = 22^h 16^m; \;\; \delta = -1°53'; \;\; \text{Mag.} = 4.0)$

Plate	Date	G.M.T.	Taken by	No. of Lines	Velocity	Quality
					km	
IB 804.........	1906 July 13	$20^h 55^m$	F	4	− 26	v.g.
1139.........	1907 Aug. 12	19 15	B	6	− 6	v.g.
2453.........	1910 July 29	18 27	B, L	8	−10	v.g.

The variable radial velocity of this star was announced by Plaskett in the *Journal of the Royal Astronomical Society of Canada*, **2**, 272, 1908.

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THE FIRST DESLANDRES' GROUP OF THE POSITIVE BAND SPECTRUM OF NITROGEN, UNDER HIGH DISPERSION

By RAYMOND T. BIRGE

This paper is a preliminary discussion of the First Deslandres' Group of the band spectrum of nitrogen, based mainly upon photographs taken by the author in the second order of a 21-foot concave grating, from λ 5000 to λ 6800. The bands are almost completely resolved into lines, and the discussion in this paper is concerned with the relations between the lines forming the three principal heads of the bands.

INTRODUCTION

The positive band spectrum of nitrogen has been the subject of a large number of investigations. Under low dispersion the apparent regularity of the bands, both in position and in appearance, is very striking. The violet end of this spectrum can easily be photographed under high dispersion, but because of the relatively low intensity in the longer wave-lengths, this portion had not previously been resolved into its component lines in a satisfactory manner. Von der Helm[1] made the latest attempt. The two objects of his investigation, with his success in accomplishing them, are stated as follows:

1. Übersicht über den gesamten Teil des in Frage kommenden Spectrums, inbesondere über die Lage der Bandenköpfe.

2. Genaueres Studium einzelner Banden.

Den ersten Teil der Aufgabe darf ich als gelöst betrachten; am zweiten bin ich leider fast völlig gescheitert.

Von der Helm gives a complete discussion of relevant previous work on nitrogen, together with the possible results of such an investigation, and the first eight pages of his article could well form the introduction to this paper.

The First Deslandres' Group of bands extends from λ 5100 out into the infra-red. Measurements on the heads of individual

[1] *Zeitschr. f. wiss. Phot.*, **8**, 405, 1910.

bands have previously been made to λ 9100. The results seem to show that the spectrum consists of a series of band groups, each of which is most intense at the center, and diminishes in intensity toward either side. Kayser's *Handbuch*[1] gives only three groups, which he calls *a*, *b*, and *c*. Other groups of longer wave-length have since been found, and it appears now that there are six groups in all, which will be designated *a* to *f* respectively.

Group *a* 1.06 μ (?)
Group *b* 9101 (?) to 7887
Group *c* 7887 to 7059
Group *d* 7059 to 6185 (Kayser's *a* group)
Group *e* 6186 to 5485 (Kayser's *b* group)
Group *f* 5632 to 5126 (Kayser's *c* group)

Group *f* is quite different from the others. It has two intensity maxima, one at λ 5200 and the other at λ 5475. This would indicate two groups, but as the spacing is the same in both, it has been customary to classify them together. This group also overlaps considerably on group *e*.

The author obtained, besides the exposures on the large grating, one on a Hilger constant deviation spectroscope, extending to λ 7650. From this point to λ 9100 we have only the measurements of Croze.[2] Coblentz,[3] in connection with other infra-red work, has recorded positions of maximum intensity at 0.546, 0.667, 0.75, 0.90, and 1.06 μ. These are very evidently the approximate positions of maximum intensity in the several band groups. The reading at 1.06 μ points to the existence at this point of another group, which we have called group *a*.

In making this investigation the author had two objects in view: (1) to determine whether or not the bands in any one group were identical; (2) to determine, in case there were any similarities, whether corresponding lines in successive bands would fit into a Deslandres' series or other arithmetical relation.

The results of the study made thus far indicate that out of the 250 or more lines composing each band, at least 50 of the strongest are related to corresponding lines in other bands, and that the relationship is approximately that expressed by Deslandres' Law:

$$v = a + b(m+c)^2$$

[1] *Handbuch der Spectroscopie*, **5**, 828.
[2] *Comptes rendus*, **150**, 860, 1910.　　　[3] *Physical Review*, **22**, 1, 1906.

where a, b, and c are constants, and m takes successive integral values.

Atmospheric nitrogen, free from oxygen, carbon dioxide, and water-vapor, was used as a source. Hence the inert gases of the atmosphere were present, but the only lines due to them which have thus far been noted are a few of the stronger argon lines of the red spectrum. There is no trace of helium $\lambda\,5876$. Traces of mercury diffused into the spectrum tube from the pressure gauge, but only the three strong lines at $\lambda\,5790$, $\lambda\,5769$, and $\lambda\,5461$ appear, the last enormously overexposed.

The nitrogen was electrically excited in a Goetze "Type C" spectrum tube. The emission from the capillary of such a tube, in a "head-on" direction, appears to be the most intense, per unit cross-section, now obtainable. The electrical excitation was furnished by the secondary of a large induction coil, the primary being run on 110 volts A.C., 1.5 amperes. The nitrogen was introduced at about 5 mm pressure and used until the pressure fell to about 1 mm, low enough to cause a slight diminution of the radiation. Refilling of the tube was necessary only once in 24 to 36 hours.

The tube was placed accurately "head-on" to the slit of the grating, 60 cm away. A double convex lens of 15 cm focus produced on the slit a sharp image of the end of the capillary, somewhat more than 1 mm in diameter. This usual arrangement was now varied by introducing, at a distance of 12 cm from the slit, a double concave cylindric lens of 12 cm focus, placed with its axis horizontal. This caused the circular image on the slit to be drawn out into a vertical line some 2 cm in length. The use of such a cylindric lens in spectrum work has been advocated by Humphreys,[1] but I know of no definite statement of the advantages and disadvantages incident to its use.

The action of the cylindric lens is greatly to reduce the vertical aperture of the cone of rays proceeding from the slit. With the particular lenses used, it is possible, with a source of light less than approximately 2 mm in diameter, to reduce the vertical aperture, at the grating, to less than the length of the grating rulings. Thus

[1] *Astrophysical Journal*, **18**, 324, 1903.

the cross-section of the cone of light at the grating, instead of being a 75-cm circle, is reduced (roughly) to an ellipse of 75 cm horizontal diameter, but with a vertical diameter of 5 cm or less. The gain in intensity of the middle point of the astigmatic image at the camera is theoretically $\frac{75 \text{ cm}}{5 \text{ cm}} = 15$. The actual increase, determined experimentally, was thirteen fold.

If now the source is made 4 mm in diameter, instead of 2, the amount of light actually striking the grating, using the cylindric lens, is scarcely increased at all. But with the ordinary arrangement, the amount would practically be doubled. Hence the advantage of the cylindric lens is proportionally decreased. For sources more than 2 cm in diameter, there is no appreciable advantage in using a cylindric lens.

The chief disadvantage attendant upon its use is the necessity of accurate adjustment. The centers of the tube, convex lens, concave lens, and slit should all lie accurately in the horizontal plane formed by the center of the grating and of the camera. With this condition fulfilled, and the cone of light falling symmetrically upon the grating, a raising or lowering of the cylindric lens of even one-tenth of a millimeter is sufficient to throw an appreciable portion of the light entirely below or above the rulings of the grating.

Because of the excess of radiation in a "head-on" direction, the illumination of the grating is far from uniform; but this is true even when the cylindric lens is not used. Such a non-uniformity is liable, however, to cause a shift of the lines of the comparison spectrum relative to those under investigation. The actual shifts found in many cases, between the iron and nitrogen lines, are believed to be due primarily to this cause.

As a comparison source I used an iron arc of the Pfund[1] type, run on 200 volts, 5 amperes, with iron and carbon electrodes. It worked in a very satisfactory manner. The exposures were made in the second order, and both the second-order and coincident third-order international iron normals were used, the measurements in the ultra-violet being those of Buisson and Fabry,[2] not yet officially adopted as standards.

No relative shift of orders could be detected on those plates where both the second- and third-order normals were present.

[1] *Astrophysical Journal*, **27**, 296, 1908.   [2] *Ibid.*, **28**, 169, 1908.

Whenever two normals fell near together and were both of suitable intensity for an accurate setting, the agreement was perfect. When one or both lines were overexposed the disagreement might be anything from 0.007 A down. This was taken to indicate that the secondary international normals, when overexposed, do not necessarily broaden symmetrically. The much greater uniformity in intensity of the normals between $\lambda\ 3500$ and $\lambda\ 4500$ thus makes them preferable for use, and this fact, coupled with the great faintness of the normals from $\lambda\ 5900$ into the red, caused the author to use only the coincident third-order normals in the region $\lambda\ 5900$ to $\lambda\ 6800$.

In order to eliminate the exceedingly strong violet bands of nitrogen, an 8 per cent solution of potassium chromate 5 mm thick was employed. The absorption of this solution sets in at about $\lambda\ 5200$ and this accounts for the rapid decrease in intensity below this point. (See Plate III.) Although the head of the $\lambda\ 3576$ band is a thousand times as intense, photographically, as that of any band under investigation, no trace of it appears on the exposures. Fluorescein was tried as an absorbent and found quite ineffective.

For the exposures from $\lambda\ 5000$ to $\lambda\ 5900$ the Cramer Instantaneous Isochromatic plates were employed, while from $\lambda\ 5800$ to $\lambda\ 6900$ both Cramer "Spectrum" and Wratten & Wainwright "A" Panchromatic were used. For the one exposure on the Hilger spectroscope, from $\lambda\ 6800$ to $\lambda\ 7700$, I used a Wratten & Wainwright "B" Panchromatic plate.

The strongest portion of the spectrum, from the photographic standpoint, is that from $\lambda\ 5700$ to $\lambda\ 5800$. The $\lambda\ 5804$ band is fully three times as intense as that at $\lambda\ 6623$, the only one which von der Helm appears to have obtained sufficiently intense for measurement. The region from $\lambda\ 5500$ to $\lambda\ 5900$ was accordingly photographed first, using $12 \times 1\frac{1}{4}$ inch plates, and the usual Rowland type of comparison shutter. All other exposures were made with $18 \times 2\frac{1}{2}$ inch plates, using a comparison shutter, mounted independent of the camera.

In making exposures several days in length, the greatest problem is a proper control of temperature. Fortunately for the author, the large grating of the University of Wisconsin is mounted inside a double-walled room, built in turn entirely inside an ordinary room.

PLATE II
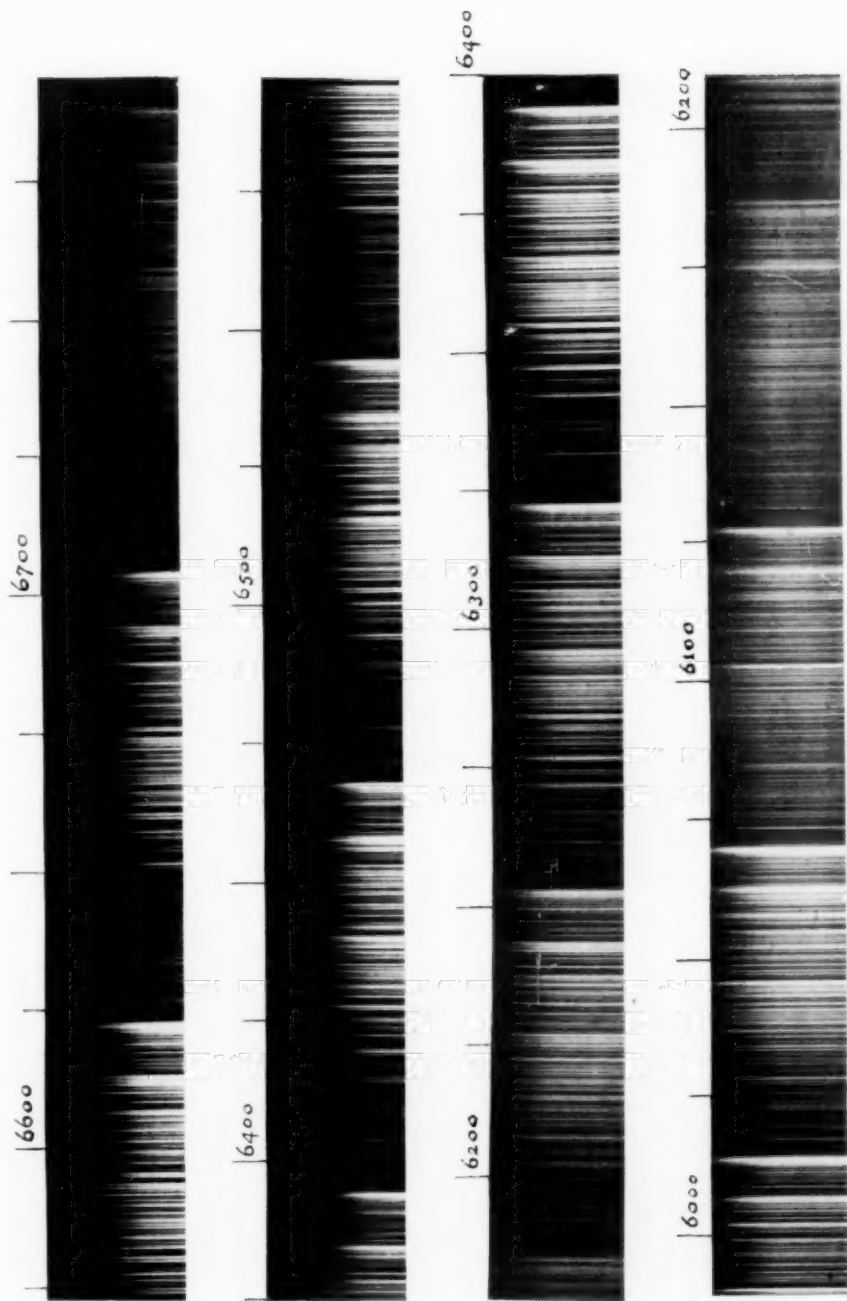
POSITIVE BAND SPECTRUM OF NITROGEN

PLATE III
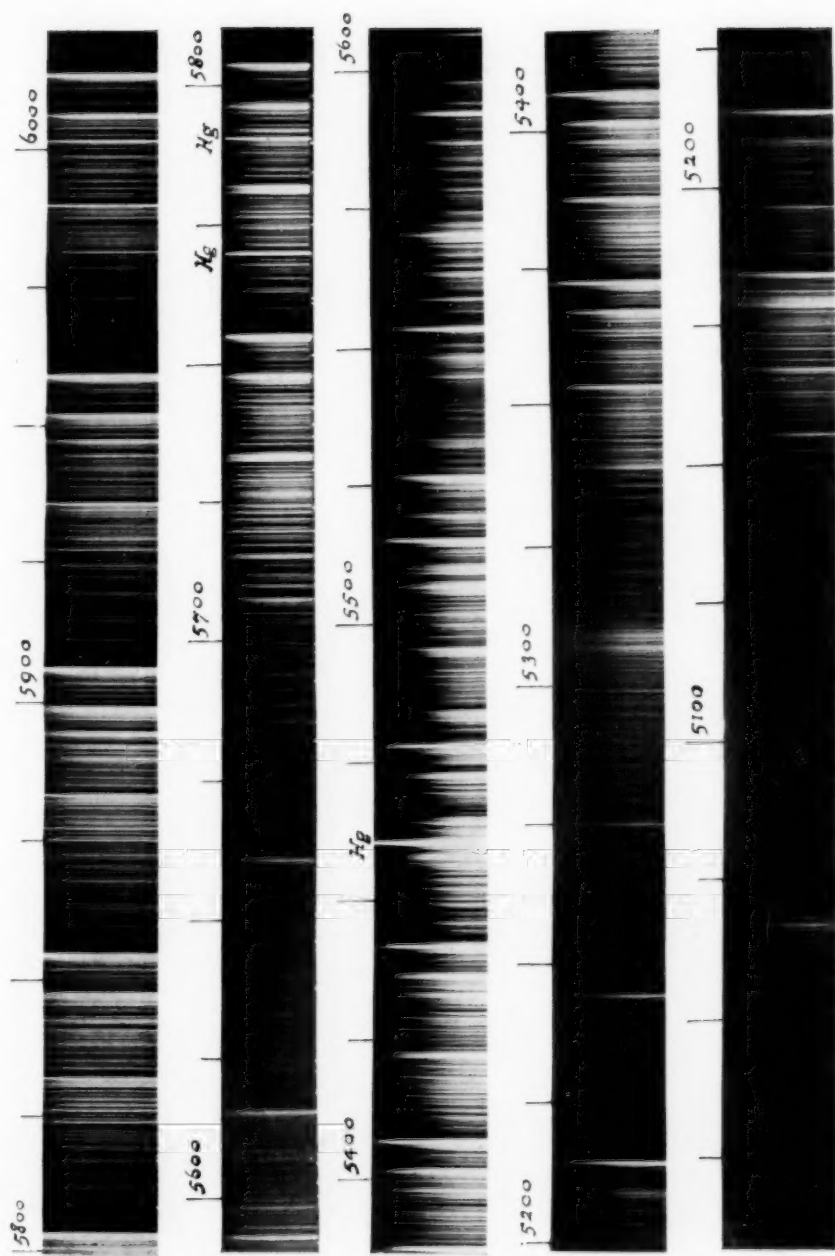
POSITIVE BAND SPECTRUM OF NITROGEN
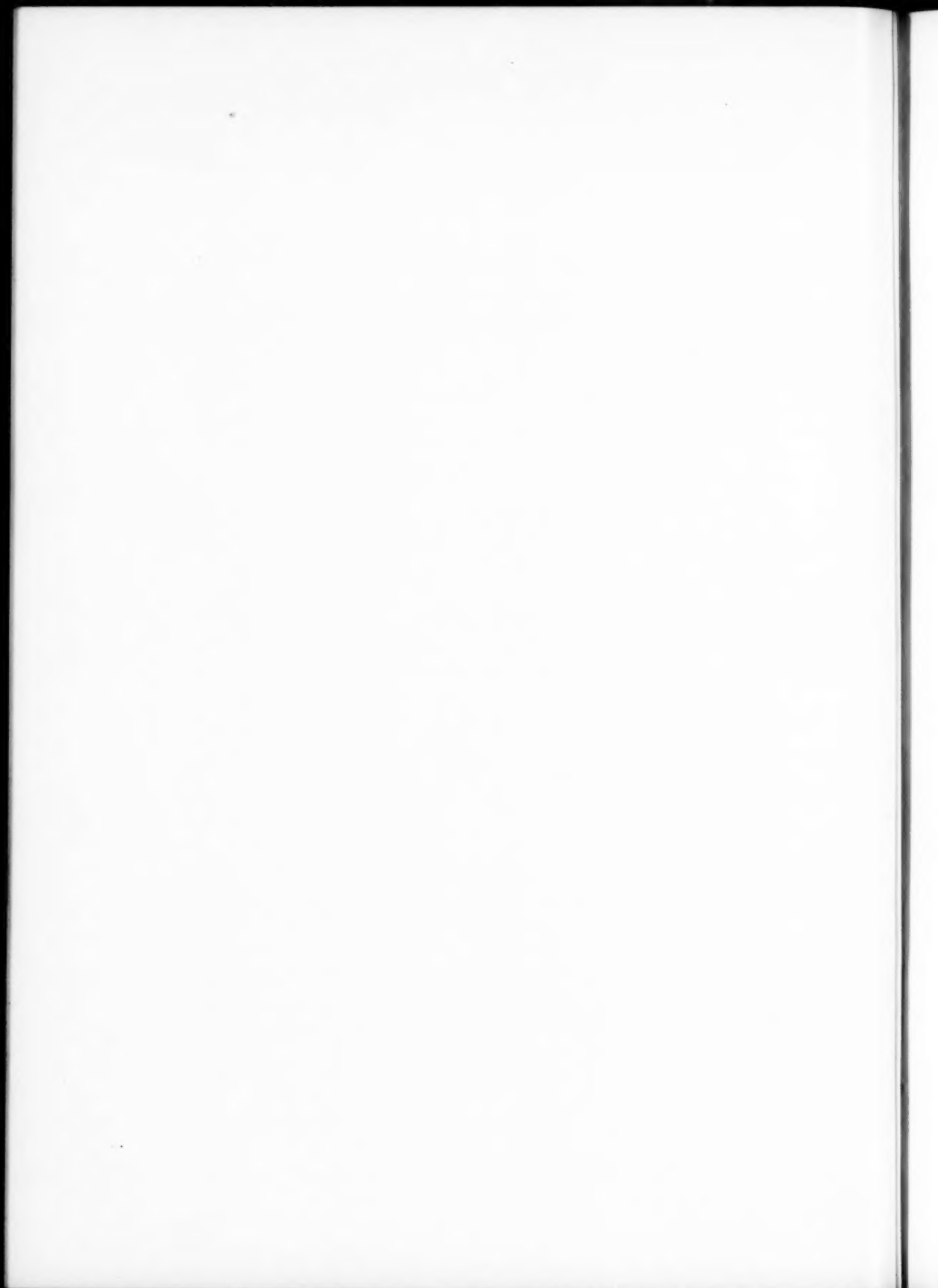
The temperature in this outer room was kept constant within a few tenths of a degree by suitable electrical heating. This enabled the temperature of the grating to be kept constant within a few hundredths of a degree. The grating temperature was read on an accurate mercury thermometer, mounted in metallic contact with the side of the grating. Other thermometers were laid in a slot in the iron beams forming the slit-grating-camera triangle. A small change of temperature in this triangle is immaterial, so long as all parts remain at an equal temperature.

For the grating, however, a constant temperature is indispensable, the change of wave-length at a given point on the camera plate being proportional, to first-order effects, to the change in the width of the grating space.[1] Holtz[2] seems to question this, and spends some time searching for other causes for the observed shift of lines with temperature. The mounting of the grating at the University of Wisconsin is such as to exclude the chief sources of error which he mentions, and it was found experimentally that the shift was exactly that computed from the change of temperature and the coefficient of expansion of the grating.

A change of $0°.01$ C. in the grating temperature will shift a line (at λ 5000) about 0.001 A. During the exposures the temperature was never allowed to leave a $0°.1$ C. range, and during any one exposure the average variation from the mean temperature varied, in different exposures, from $0°.015$ to $0°.035$ C. The broadening of the lines was thus always less than 0.01 A.

Not only the temperature, but the barometric pressure as well, causes a shift of the spectrum. A change of 1 mm in pressure will shift the lines 0.002 A. With frequent total pressure variations of 2 cm, sufficient to cause a 0.04 A broadening of the lines, it becomes necessary to eliminate this change also. This was done by arbitrarily changing the temperature. A 1 cm rise of pressure is compensated by a $0°.15$ lowering of temperature. The mean temperature mentioned above, which I endeavored to hold constant, refers to the initial temperature, properly corrected for subsequent change in barometric pressure.

The time of exposure varied from 66 to 120 hours. The slit-

[1] See Baly, *Spectroscopy*, p. 241, 1912 edition.

[2] *Zeitschr. f. Wiss. Phot.*, **12**, 101, 1913.

width varied from 0.01 to 0.04 mm, being usually 0.02 mm. The theoretical resolving power of the grating (a 6-inch, 14,438-line grating), for the slit-width used, was actually obtained on all exposures except those in the red where, in the second order, the grating has a somewhat poorer definition.

The spectrum·was photographed on eight different plates, two for each region. These regions were (1) λ 6900–λ 6300, (2) λ 6400–λ 5800, (3) λ 5900–λ 5500, (4) λ 5600–λ 5000. For regions (1) and (2), one was a Cramer plate, the duplicate a Wratten & Wainwright plate. No plates were exact duplicates, as the slit-width and time of exposure were varied. One 85-minute exposure was made on a Hilger spectroscope, for the region λ 6800–λ 7700. A one-minute exposure is sufficient, on this instrument, for the shorter wavelengths. The spectroscope was calibrated with the argon spectrum, and the readings obtained for nitrogen are probably correct to 1 A. All of the plates obtained with the large grating are usable save one in the λ 6300–λ 6900 region which dried very unevenly. The duplicate plate, however, is the best that I have, and the readings obtained from it are believed to be as trustworthy as those in any portion of the spectrum.

The work that has thus far been completed is as follows:

1. The lines in the immediate vicinity of the three conspicuous "heads" of each band have been measured, and their wave-lengths computed, on all plates.

2. The regions λ 5500–λ 5900 and λ 6300–λ 6900 have been completely measured and computed.

There are about 6400 lines between λ 5000 and λ 6800, and 274 in the λ 6623 band, in which von der Helm measured 119. There appear to be fully as many in all the other bands, although in most cases the number actually measured is much less, owing to the smaller intensity and shorter length of the bands.

The measurements were made on a 55-cm Geneva dividing engine. The screw was carefully calibrated by the author and is believed to have no unknown errors greater than 0.002 mm. In order to test the evenness of drying of the plates, the international secondary standards were first corrected for non-normality of the dispersion and errors of the screw, and were then fitted as nearly as possible to a linear scale. Only standards of suitable intensity

were used, those overexposed being evidently untrustworthy. In the case of one plate in the λ 6300–λ 6900 region, the average deviation of all the normals from a linear scale was less than 0.002 A. This was taken to indicate that the screw had been correctly calibrated. On other plates there was a general drift from such a linear scale, very evidently due to uneven drying. It seldom exceeded 0.015 A and by drawing a smooth curve through the plotted readings of the normals, the correction for this was easily made.

When the wave-length determinations of one plate were compared with those of a duplicate plate, there generally appeared a constant difference between them. This difference varied from 0.01 A to 0.04 A on different sets of plates. It was considered due to the uneven illumination of the grating, as already explained. Fortunately, however, we have interferometer measurements of the three mercury lines present on my plates. By means of the ghosts and satellites of these lines, it was possible to determine their position with great accuracy, in spite of their overexposure. This settled the absolute wave-lengths from λ 5100 to λ 5900. One plate in each of the other two regions was then found to agree perfectly in the overlapping portions. I thus had a full set of plates in complete agreement, and the duplicate plates were then given the proper constant correction to make them also agree.

The values of the wave-length of any one line, as determined on different plates, then seldom differed by more than 0.01 A. Several settings were made on each line, and as the nitrogen lines are fairly sharp, the average experimental error of setting scarcely exceeds 0.003 A. It is hoped, therefore, that the relative error of all save very faint or hazy lines is less than 0.005 A, and that the absolute wave-lengths are in general correct to 0.01 A.

Table I gives the wave-lengths of 872 lines forming the three principal heads of the bands. The lines in the vicinity of all the heads given by von der Helm were measured, although in several cases there is no real head present. Several other heads not given by von der Helm were noted and measured. These so-called "heads" are caused by the proximity of several heavy lines, accompanied by more or less continuous radiation. The measurements, in all cases, cover this region of continuous radiation, which is indicated in the table by braces. Frequently the haze is due

merely to the scattering of light in the photographic film, but in most cases it is apparently a true radiation.

The three main heads of a band, out of the five that appear with low dispersion, are designated I, II, and IV. The bands themselves are designated in two ways: first, by the division into groups (*a* to *f*), the individual bands of each group, from red to violet, being designated by Arabic numerals; the second method of designation is that proposed by Cuthbertson[1] and formulated mathematically by Deslandres.[2] In this arrangement the position of the first head of each of the entire set of 57 bands is given as a function of two independent parameters, *p* and *n*. The value of these parameters, for each band, is given immediately below the designation of the band according to the first arrangement. The first integer refers to the value of *p*, the second to *n*—the values being those of Deslandres.[3]

The three columns in the table are:

(1) Intensity; lines marked " ? " are so faint as to preclude an accurate determination of wave-length; (2) wave-length—on the International System (I.A.), at 15° C., 760 mm; (3) character of the line. In this regard the following abbreviations are used:

s.,   especially sharp.
b.,   broad.
b.d.,   broad, probably double.
d.,   certainly double.
h.,   hazy.
h.r.,   haze on the red side (due to one or more fainter components on that side).
h.v.,   haze on violet side.
n.s.,   a non-symmetric line due to two or more components of unequal intensity. The setting was made on the center of gravity of the system.
k.,   the line at which a "head" apparently starts.
a.,   argon.

Von der Helm's value for the wave-length *in air* for the general position of the head, together with the frequency *in vacuo*, is given to the right of the designation of the head.

[1] *Phil. Mag.* (6), **3**, 348, 1902.   [2] *Comptes rendus*, **134**, 747, 1902.
[3] See Baly, *Spectroscopy*, p. 620, 1912 edition.

TABLE I

1	2	3
{I d 4	{ 6787.91	
{48–53	{ 17,728.1	
1	6788.614	k.
{1	.243	
{1	.101	
{1	87.970	
2	.834	
2	.712	
1	.515	
2	.270	
{II d 4	{ 6778.35	
{48–53	{ 14,748.9	
1	6779.972	
1	78.821	
{1	.623	k.
{2	.448	
2	77.949	
1	.538	
1	.288	
2	76.874	
2	.661	
{IV d 4	{ 6758.98	
{48–53	{ 14,791.2	
2	6759.243	
2	58.054	k.
{1	57.807	
{1	.665	
2	.355	
1	.067	
{1	56.721	
{1	.611	
1	.325	
4	55.948	
{I d 5	{ 6704.45	
{47–52	{ 14,911.4	
{3	6704.755	k.
2	.634	
1	.514	
3	.363	
3	.132	
{3	03.879	h.d.
3	.630	
3	.376	
4	.227	

1	2	3
{II d 5	{ 6694.95	
{47–52	{ 14,932.6	
4	6694.911	k.
2	.775	
1	.553	
2	.391	
3	.226	{n.s. h.v.
2	93.774	
1	.610	b.
1	.474	
{3	.367	
{3	.242	
3	92.849	
{IV d 5	{ 6675.01	
{47–52	{ 14,977.2	
{3	6674.908	k.b.
2	.634	b.
4	.236	
2	.074	
2	73.817	
3	.615	
1	.446	
{4	72.954	
{4	.852	
{I d 6	{ 6623.534	
{46–51	{ 15,093.6	
{4	6623.574	k.b.
2	.417	
2	.281	
{4	.120	b.h.
4	22.915	
2	.795	
{5	.658	
3	.395	b.h.
{4	.130	n.s.
{4	21.071	s.
3	.838	
{II d 6	{ 6614.023	
{46–51	{ 15,115.0	
6	6614.031	k.
{2	13.789	h.
{4	.678	
3	.514	

1	2	3
3	6613.188	h.r.
4	.061	
{2	12.878	h.
{2	.759	h.
{4	.524	
{4	.244	s.
1	.001	
3	11.722	
2	.603	
{IV d 6	{ 6594.425	
{46–51	{ 15,160.1	
{5	6594.418	k.b.
4	.175	
6	93.739	
{2	.598	h.v.
5	.155	
4	92.568	
6	.423	
3	91.936	
2	.781	
{I d 7	{ 6544.81	
{45–50	{ 15,275.2	
{3	6544.881	k.
{3	.716	
1	.598	
5	.432	h.
4	.237	
2	.095	
6	43.942	
{2	.714	
{2	.616	
4	.460	
4	.251	
{II d 7	{ 6535.50	
{45–50	{ 15,296.8	
5	6535.655	s.
6	.110	d.
3	34.924	
4	.627	
3	.482	
4	.188	h.r.
3	.028	
4	33.754	{h.r. h.v.
1	.305	
4	.127	b.

TABLE I—*Continued*

1	2	3
$\begin{cases}\text{IV } d\ 7 \\ 45\text{–}50\end{cases}$	$\begin{cases}6516.44 \\ 15{,}341.6\end{cases}$	
2	6516.610	
4	.403	h.r.k.
2	.256	h.
2	.166	h.
5	15.897	
6	.759	
1	.586	
2	.407	
4	.181	
6	14.687	
5	.459	
$\begin{cases}\text{I } d\ 8 \\ 44\text{–}49\end{cases}$	$\begin{cases}6468.53 \\ 15{,}455.3\end{cases}$	
4	6468.597	k.b.
4	.438	
5	.144	h.r. h.v.
4	67.951	
2	.802	
6	.634	
2	.416	
3	.280	
3	.142	
4	66.913	
2	.809	
2	.588	
5	.442	b.
$\begin{cases}\text{II } d\ 8 \\ 44\text{–}49\end{cases}$	$\begin{cases}6459.04 \\ 15{,}478.1\end{cases}$	
6	6459.673	
2	.181	
5	58.884	k.
2	.726	h.
3	.503	
2	.385	
4	.261	
2	.128	
2	57.810	
4	.669	
2	.460	h.d.
3	.086	
$\begin{cases}\text{IV } d\ 8 \\ 44\text{–}49\end{cases}$	$\begin{cases}6440.80 \\ 15{,}521.8\end{cases}$	

1	2	3
2	6441.134	
4	40.768	k.h.d.
3	.563	
2	.392	
4	.150	
2	39.992	
3	.590	
4	.270	
4	.168	
5	38.887	
$\begin{cases}\text{I } d\ 9 \\ 43\text{–}48\end{cases}$	$\begin{cases}6394.45 \\ 15{,}634.2\end{cases}$	
2	6394.628	k.
3	.442	
1	.284	
2	.122	
3	93.991	h.v.
1	.854	
5	.636	
$\begin{cases}\text{II } d\ 9 \\ 43\text{–}48\end{cases}$	$\begin{cases}6384.93 \\ 15{,}657.6\end{cases}$	
2	6386.096	
4	85.978	
2	.503	
3	.026	k.
3	84.900	
4	.627	d.
3	.451	b.
2	.322	
2	.081	
4	83.887	b.
$\begin{cases}\text{IV } d\ 9 \\ 43\text{–}48\end{cases}$	$\begin{cases}6367.55 \\ 15{,}700.3\end{cases}$	
4	6367.416	h.r.
2	.165	
4	66.808	
2	.252	
3	.107	
3	65.913	
3	.564	
$\begin{cases}\text{I } d\ 10 \\ 42\text{–}47\end{cases}$	$\begin{cases}6322.73 \\ 15{,}811.6\end{cases}$	

1	2	3
2	6322.816	k.h.
?	.708	
2	.594	
1	.462	
1	.386	
2	.280	
1	.162	
2	.003	n.s.
4	21.797	n.s.
$\begin{cases}\text{II } d\ 10 \\ 42\text{–}47\end{cases}$	$\begin{cases}6313.20 \\ 15{,}835.5\end{cases}$	
4	6314.420	
2	13.287	k.
2	.185	h.
2	.069	
1	12.957	
2	.720	h.
2	.626	h.
1	.214	h.
4	11.659	
$\begin{cases}\text{IV } d\ 10 \\ 42\text{–}47\end{cases}$	$\begin{cases}6296.03 \\ 15{,}878.8\end{cases}$	
3	6296.212	b.h.
1	.026	n.s.
?	95.805	
3	.606	
3	.077	
$\begin{cases}\text{I } d\ 11 \\ 41\text{–}46\end{cases}$	$\begin{cases}6252.81 \\ 15{,}988.4\end{cases}$	
1	6253.001	b.k.
2	52.806	
?	.670	h.
?	.587	h.
1	.494	
1	.377	
1	.226	b.
$\begin{cases}\text{II } d\ 11 \\ 41\text{–}46\end{cases}$	$\begin{cases}6243.51 \\ 16{,}012.2\end{cases}$	
?	6243.688	
2	.581	s.h.v.
2	.297	
1	42.944	h.r.
1	.744	

TABLE I—*Continued*

1	2	3
1	6242.601	
?	.402	h.
3	.199	
{IV d 11	{ 6227.00	
{41–46	{ 16,054.7	
{2	6227.096	h.k.
{3	26.978	b.h.
{1	.790	h.
2	.416	
1	.079	h.
{I e 1	{ 6185.44	
{49–53	{ 16,162.6	
?	6187.022	h.r.
1	86.733	k.
2	.297	b.h.
1	85.874	
1	.570	b.
{I d 12		
{40–45		
{1	6185.224	
{1	.127	
?	84.937	d.
1	.750	b.
1	.516	b.
1	.313	b.
1	.120	s.
1	.012	s.
?	83.861	
{II e 1 ?		
{49–53		
2	6178.549	
2	.170	
?	77.574	
?	.385	
2	.259	
?	76.809	
{II d 12	{ 6175.32	
{40–45	{ 16,189.1	

1	2	3
{2	6176.126	k.
{1	75.967	
{?	.791	
{?	.654	
1	.523	
1	.392	
1	.251	
1	.112	
2	74.874	
{3	.612	
{IV e 1 ?		
{49–53		
2	6161.648	
1	.276	
1	.131	
?	60.797	
?	.580	
1	.395	b.
{IV d 12	{ 6160.43	
{40–45	{ 16,228.2	
{1	6159.844	h.k.
{2	.692	h.
?	.443	
{2	.114	
1	58.952	h.
1	.581	h.
{2	.433	
2	57.960	d.
{I e 2	{ 6127.23	
{48–52	{ 16,316.1	
2	6127.374	h.v.k.
1	.208	
1	.087	
4	26.944	
1	.752	
2	.471	
{II e 2	{ 6119.79	
{48–52	{ 16,336.0	
2	6120.602	
2	.273	h.v.
{1	19.849	k.
{2	.720	h.v.

1	2	3
3	6119.454	
1	.312	b.
2	.129	b.
1	18.993	
{IV e 2	{ 6102.60	
{48–52	{ 16,382.0	
{2	6102.736	k.
{1	.514	
{3	.381	
{1	.259	
2	01.537	
{I e 3	{ 6069.60	
{47–51	{ 16,471.0	
{5	6069.663	b.k.
{2	.463	
{5	.280	h.v.
{3	.174	
{2	.034	h.v.
5	68.704	
1	.506	
3	.382	
2	.250	
{II e 3	{ 6062.44	
{47–51	{ 16,490.5	
3	6062.563	
2	.414	
2	.190	h.r.
6	61.984	b.
3	.500	h.r.
{IV e 3	{ 6045.55	
{47–51	{ 16,536.6	
{3	6045.484	k.
{3	.407	
{6	.173	h.r.
{1	44.970	d.
{2	.808	
4	.319	h.r. h.v.
{I e 4	{ 6013.60	
{46–50	{ 16,624.4	

TABLE I—*Continued*

1	2	3
5	6013.575	k.h.r.
3	.335	
6	.195	s.
4	.030	
2	12.907	
3	.772	b.
4	.567	
II e 4 46–50	6006.34 16,644.5	
4	6006.477	
2	.341	
5	.118	
4	05.967	
3	.834	
4	.645	b.
IV e 4 46–50	5990.01 16,689.9	
4	5989.812	n.s.k.
5	.636	s.
3	.519	
1	.324	
2	.179	
4	88.702	
I e 5 45–49	5959.25 16,776.0	
2	5959.220	
5	.053	h.v.k.
3	58.767	
5	.656	b.
2	.548	
3	.447	
2	.342	
3	.206	
II e 5 45–49	5952.02 16,796.4	
4	5951.935	h.k.
5	.782	
4	.617	
3	.440	
1	.332	
2	.115	
2	.010	

1	2	3
IV e 5 45–49	5935.95 16,841.9	
3	5935.920	
3	.740	k.
4	.660	h.r.
3	.553	
5	.426	
1	.231	
3	.092	
1	34.925	
3	.774	
5	.627	
I e 6 44–48	5906.24 16,926.6	
5	5906.010	b.k.
1	05.900	
?	.792	
6	.672	h.v.b.
2	.503	
3	.368	
2	.285	
2	.126	
3	04.948	s.
3	.852	
II e 6 44–48	5899.19 16,946.7	
2	5899.070	
5	98.930	n.s.k.
2	.631	
2	.523	
3	.401	b.
?	.252	
1	.148	
3	.040	b.
3	97.874	
IV e 6 44–48	5883.49 16,992.0	
2	5883.517	
2	.446	
5	.146	k.
5	.029	
1	82.917	
3	.807	

1	2	3
2	5882.615	h.
3	.479	h.
3	.320	
6	.016	h.v. / h.r.
I e 7 43–47	5854.69 17,075.6	
5	5854.404	b.d.k.
2	.253	
2	.168	
7	.032	b.
3	53.873	
4	.666	h.r.
2	.462	
4	.286	
2	.168	s.
II e 7 43–47	5847.67 17,096.1	
?	5847.740	
5	.518	s.k.
1	.405	
2	.300	
2	.243	
1	.120	
2	.040	
6	46.815	b.h.
1	.578	h.r.
5	.477	h.r.
4	.111	h.
IV e 7 43–47	5832.29 17,141.2	
3	5832.054	k.
6	31.881	
1	.707	
3	.597	
1	.405	b.
3	.274	
?	.119	
2	30.941	
4	.839	
I e 8 42–46	5804.28 17,224.0	

TABLE I—Continued

1	2	3
6	5804.135	b.k.
1	03.978	
8	.776	d.
3	.590	
1	.482	
4	.367	
2	.195	
2	.123	
3	02.980	
3	.858	
II e 8 / 42–46	5797.23 / 17,244.9	
5	5797.451	h.r.
?	.308	
6	.062	
1	96.969	
3	.830	
?	.707	
5	.602	
2	.513	
2	.412	b.
2	.299	
IV e 8 / 42–46	5782.23 / 17,289.1	
5	5782.307	
8	.059	b.k.
1	81.943	
1	.849	
5	.740	
3	.566	
3	.426	
4	.239	
6	80.984	
4	.836	
I e 9 / 41–45	5755.20 / 17,370.8	
2	5755.415	
4	.188	h.k.
?	.074	
3	54.983	
5	.862	
5	.746	
4	.616	
1	.498	
6	.366	
2	.195	h.v.

1	2	3
II e 9 / 41–45	5748.18 / 17,392.0	
5	5748.664	
1	.289	
5	.101	k.
1	.008	
3	47.883	b.
8	.650	b.
6	.408	s.
2	.178	
2	.040	
1	46.939	h.v.
8	.615	
1	.487	
1	.250	h.
2	.149	
IV e 9 / 41–45	5733.68 / 17,436.0	
4	5733.830	
4	.555	k.
4	.450	
1	.369	
2	.276	
4	.159	s.
1	32.992	
3	.830	b.
1	.703	
9	.462	
1	.279	
2	31.921	
I e 10 / 40–44	5707.49 / 17,516.0	
3	5707.580	k ?
2	.462	h.
1	.301	
2	.131	b.
?	06.944	
?	.840	
3	.597	h.
II e 10 / 40–44	5699.50 / 17,540.6	
3	5701.358	
2	00.939	
?	.883	

1	2	3
2	5700.637	
2	.512	
?	5699.830	
IV e 10 / 40–44	5685.56 / 17,583.7	
?	5685.496	
1	.199	
1	.058	b.
I e 11 / 39–43	5660.54 / 17,661.4	
1	5660.842	k.
?	.590	
?	.421	
1	.331	
1	.240	
1	.115	
2	59.954	b.
II e 11 / 39–43	5653.31 / 17,683.9	
?	5653.504	s.h.r.
?	.170	
?	52.676	
?	51.841	s.
IV e 11 / 39–43	5639.17 / 17,728.2	
?	5639.349	
?	38.868	
?	37.899	
?	.775	
I f 1 / 49–52		

Haze 5632.754 to 5632.361
Very faint lines to 5627.750

1	2	3
II f 1 / 49–52	5622.97 / 17,779.3	
?	5622.610	

TABLE I—*Continued*

1	2	3	1	2	3	1	2	3
{I *e* 12	{ 5615.00		{IV *f* 2			2	5553.996	
{38–42	{ 17,804.5		{48–51			2	.924	
						5	.730	b.h.k.
1	5615.318	k.				2	.600	
?	.230		2	5573.126	s.	2	.491	
2	.032	b.	2	72.894		6	.362	b.h.v.
1	14.900	b.	1	.737		4	.204	b.h.
1	.779		2	.551	a.	3	52.962	
1	.695		2	.347	h.r.k.			
1	.573		3	.247		{II *f* 3	{ 5548.40	
1	.387	d.	2	.039		{47–50	{ 18,018.2	
?	.230		1	71.881				
			3	.780		2	5548.825	
{II *e* 12	{ 5607.73		1	.638		3	.711	
{38–42	{ 17,827.6		2	.410	s.	1	.600	
			2	.320	s.	1	.515	
1	5608.214	k.				3	.390	
?	.094		{I *e* 13	{ 5570.60		2	.242	
?	07.687		{37–41	{ 17,946.5		1	.137	h.
?	.375							
			3	5570.777	k.h.	{IV *f* 3	{5533.46	
{I *f* 2	{ 5592.57		1	.679		{47–50	{ 18,066.9	
{48–51	{ 17,876.0		4	.501	b.h.			
			2	.354		5	5533.504	
?	5593.014		2	.201	d.	3	.406	
3	92.881	h.v.k.	2	.013	h.v.	1	.238	
1	.657		2	69.850		1	.164	
4	.514		2	.786		1	.041	
2	.364	h.				4	32.955	h.r.
2	.283	h.	{II *e* 13	{ 5563.48		?	.824	
2	.130		{37–41	{ 17,969.4		?	.707	
1	.013					4	.599	h.r.
?	91.888		2	5563.704		1	.473	h.
?	.768		1	.571		2	.349	s.
1	.586		3	.244	b.k.	2	.251	s.
			?	.124		2	.146	s.
{II *f* 2			1	.038				
{48–51			?	62.937		{I *e* 14	{ 5526.84	
			?	.828		{36–40	{ 18,088.6	
2	5588.081	k.	?	.696				
?	87.989		?	.612		2	5527.150	k.h.
1	.883		1	.405	b.h.	4	.027	
2	.742	b.n.s.				5	26.835	s.
3	.531	h.	{I *f* 3	{ 5553.63		4	.707	
3	.440	h.	{47–50	{ 18,001.2		2	.610	
1	.190					4	.508	
2	.064		2	5554.258		4	.356	
?	86.620		2	.130		3	.188	
2	.490							
2	.330					{II *e* 14	{ 5520.11	
						{36–40	{ 18,110.6	

TABLE I—*Continued*

1	2	3
3	5519.682	
5	.592	
?	.475	
1	.398	
2	.268	
I f 4 / 46–49	5515.54 / 18,125.6	
1	5515.758	
8	.594	k.b.
7	.451	b.
5	.239	b.h.
5	.178	
3	.081	
2	14.953	b.
3	.777	h.r.
3	.636	b.h.
2	.496	
2	.325	b.d.
3	.190	d.
II f 4 / 46–49	5510.55 / 18,142.3	
4	5511.096	h.r.
?	10.998	
?	.929	
4	.835	
4	.638	d.h.v.
1	.488	b.
5	.278	b.
?	.177	
?	.109	
?	.044	
1	09.936	
2	.847	
3	.736	s.
2	.545	s.
4	.434	
3	.316	s.
IV f 4 / 46–49	5495.42 / 18,192.0	
1	5495.997	
4	.885	a.(?)
2	.763	
3	.692	s.
3	.593	s.
2	.430	h.
1	.343	h.
4	.159	h.r.

1	2	3
I e 15 / 35–39		
5	5484.730	
2	.593	
3	.488	
5	.338	b.k.
3	.225	
3	.094	
3	.018	
3	83.896	
4	.686	h.
3	.476	s.
I f 5 / 45–48	5478.73 / 18,247.4	
1	5478.575	
7	.471	n.s.k.
3	.266	
6	.124	b.
4	.007	
2	77.912	
2	.804	
2	.656	
2	.601	
II f 5 / 45–48	5473.16 / 18,266.0	
2	5473.524	k.
2	.468	
4	.313	
3	.190	h.
4	.056	h.
5	72.936	
4	.590	h.v.
5	.485	
5	.316	
1	.228	
2	.131	
IV f 5 / 45–48	5458.22 / 18,315.9	

1	2	3
4	5458.879	h.k.
3	.759	n.s.
2	.630	h.
2	.554	h.
2	.451	
3	.353	s.
3	.273	s.
3	.139	
2	.008	
3	57.903	
2	.797	
1	.697	
4	.587	
I f 6 / 44–47	5442.25 / 18,369.7	
2	5442.498	
2	.416	
6	.325	k.
4	.276	
4	.200	
4	.104	
7	41.981	
5	.932	
4	.833	
3	.751	
4	.640	s.
II f 6 / 44–47	5437.03 / 18,387.3	
5	5437.328	k.d.
2	.197	s.
2	.027	
2	36.957	b.
2	.894	b.
3	.801	
3	.621	
5	.518	
3	.399	s.
IV f 6 / 44–47	5424.17 / 81,430.9	
1	5423.423	
2	.319	
2	.209	

TABLE I—*Continued*

1	2	3
{4	5423.014	
{4	22.922	
{3	.771	
{2	.688	
{6	.535	b.
{4	.369	
{6	.187	b.
1	.035	
{I ƒ 7	{ 5407.08	
{43–46	{ 18,489.2	
2	5407.575	
3	.411	b.h.
6	.129	h.r.k.
1	.049	
3	06.979	
3	.876	
6	.785	
4	.730	
3	.590	h.v.
2	.528	
{II ƒ 7	{ 5401.83	
{43–46	{ 18,507.1	
4	5401.943	b.
3	.810	k.
4	.689	b.
4	.565	
5	.450	
3	.340	
2	.189	
3	.115	
1	.024	
2	00.924	
6	.654	
{IV ƒ 7	{ 5387.82	
{43–46	{ 18,555.3	
2	5388.490	
2	.353	
?	.236	
?	.172	
5	.087	k.
5	.002	
4	87.856	
4	.767	
5	.604	
3	.518	
4	.447	h.
3	.385	h.
3	.283	

1	2	3
1	5387.192	h.
2	.050	
{I ƒ 8	{ 5372.78	
{42–45	{ 18,607.2	
2	5373.189	
2	72.976	
7	.820	h.k.
3	.727	
7	.496	b.
3	.390	
3	.223	b.h.
{II ƒ 8	{ 5367.41	
{42–45	{ 18,625.8	
4	5367.782	b.
3	.654	b.
3	.527	k.
3	.420	
5	.325	
3	.236	
2	.132	
3	.081	
1	66.973	d.
1	.853	
2	.760	
{IV ƒ 8	{ 5353.73	
{42–45	{ 18,673.4	
3	5354.073	k.
3	53.992	
3	.872	b.
1	.768	
6	.608	n.s.
2	.493	
{I ƒ 9	{ 5339.27	
{41–44	{ 18,724.0	
2	5339.432	h.}k.
1	.393	
3	.294	
1	.192	
1	.089	
?	38.920	d.
{II ƒ 9		
{41–44		

1	2	3
2	5334.318	d.
?	.165	
?	.026	b.h.
?	33.878	
3	.762	
{IV ƒ 9	{ 5323.40	
{41–44	{ 18,779.8	
1	5323.821	
?	.523	
?	.411	
?	22.983	b.h.
?	.770	b.h.
1	.615	n.s.
?	.483	b.h.
{I ƒ 10	{ 5306.22	
{40–43	{ 18,840.0	
{1	5307.072	
{1	.004	
2	06.859	k.
2	.529	
1	.146	
2	05.980	
1	.696	
2	.413	
{II ƒ 10		
{40–43		
2	5302.604	
1	.170	
?	.610	
?	.524	
1	.885	d.
{I ƒ 11	{ 5274.35	
{39–42	{ 18,953.9	
1	5275.072	k.
?	74.891	
2	.777	h.h.v.
?	.347	
{I ƒ 12	{ 5244.05	
{38–41	{ 19,063.9	
{1	5244.071	k.
{2	43.782	

TABLE I—*Continued*

1	2	3
{I *f* 13 / 37–40	{ 5213.04 / 19,177.4	
{2 / 3 / 1	5213.808 / .540 / .021	k. / h.v.
{II *f* 13 / 37–40	{ 5207.79 / 19,196.7	
2	5209.009	s.
2	08.669	b.h.
1	.503	h.
3	.300	
2	.157	
1	07.885	
3	.558	h.r.
1	.262	h.
2	06.894	s.
2	.800	
4	.024	s.
{IV *f* 13 / 37–40		
{3 / ?	5196.370 / .196	h.v.

1	2	3
1	5196.080	
2	95.967	
1	.679	
2	.443	
2	.346	
2	94.854	
{I *f* 14 / 36–39	{ 5183.84 / 19,285.4	
3	5184.237	b.k.
4	83.970	b.
1	.843	
1	.734	
1	.679	
1	.550	
2	.445	
2	.394	
{II *f* 14 / 36–39		
4	5179.876	n.s.
3	.567	
1	.460	
2	.322	
3	.217	
2	.072	

1	2	3
1	5178.940	
{IV *f* 14 / 36–39		
2	5167.060	k.
2	66.992	
1	.872	
1	.766	
2	.676	s.
1	.410	
1	.309	
2	.181	
2	.058	
{I *f* 15 / 35–38		
2	5155.323	k.
2	.201	
1	.095	
{I *f* 16 / 34–37		
{1 / ?	5126.806 / .726	k.

The following table (Table II) gives the measurements of all conspicuous lines, or groups of lines, in the bands extending from λ 6800 to λ 7650, as taken on the Hilger spectroscope. The probable error is 1 A.

The three main heads are designated as before, using only the first method of grouping. The columns are: (1) wave-length *in air;* (2) frequency *in vacuo;* (3) designation of head.

Table III gives Croze's measurements of the first head of the bands from λ 7600 to λ 9100. The probable error is several angstroms.

DISCUSSION

The discussion naturally falls into three sections: (1) a brief sketch of the two methods previously proposed for grouping the heads of the nitrogen bands; (2) a quantitative test of the comparative validity of the two methods, based upon the data given

TABLE II

1	2	3	1	2	3	1	2	3
7624.8	13,111.5	I c 3	7261.3	13,768.1	II c 6	7072.8	14,134.8	
7613.5	13,131.2	II c 3	⎰7254.3	13,781.3		7059.6	14,161.2	⎱I c 8
7589.4	13,172.9	IV c 3	⎱7250.0	13,789.4				⎰I d 1
7505.6	13,319.9	I c 4	7239.9	13,808.5	IV c 6	7048.5	14,183.7	II c 8
7492.2	13,343.7	II c 4	7233.1	13,821.6		7040.4	14,200.2	
7469.6	13,383.9	IV c 4	7228.5	13,830.9		7028.2	14,224.7	IV c 8
7445.3	13,427.6		7221.6	13,843.7		7016.5	14,248.3	
7404.8	13,501.0		7214.0	13,858.3		7001.4	14,279.1	
7386.1	13,535.3	I c 5	7205.2	13,875.3		6991.5	14,299.3	
7375.6	13,554.5	II c 5	7197.3	13,890.5		6978.6	14,325.9	
⎰7366.8	13,570.7		7181.0	13,921.8		6968.0	14,347.6	I d 2
⎱7363.0	13,577.7		7165.0	13,953.0	I c 7	6956.0	14,372.3	II d 2
7352.8	13,596.5	IV c 5	7153.0	13,976.2	II c 7	6946.0	14,392.8	
7345.6	13,609.9		⎰7145.2	13,992.0		6938.8	14,408.1	IV d 2
7338.3	13,623.3		⎱7142.2	13,997.5		6929.5	14,427.2	
7331.0	13,637.0		7132.2	14,017.2	IV c 7	6921.6	14,443.8	
7323.4	13,651.3		7125.0	14,031.4		6906.0	14,476.1	
7315.3	13,666.3		7120.0	14,041.2		6896.6	14,497.2	
7307.7	13,680.6		7112.3	14,056.5		6875.5	14,540.5	I d 3
7291.2	13,711.5		7099.3	14,082.0		6865.2	14,562.3	II d 3
7274.0	13,743.8	I c 6	7090.9	14,098.7				

TABLE III

1	2	3	1	2	3
9101	10,985	I b 1	8043	12,430	I b 7
8903	11,229	I b 2	7887	12,676	⎰I b 8
8707	11,482	I b 3			⎱I c 1
8541	11,705	I b 4	7742	12,913	II c 2 (?)
8369	11,946	I b 5	7628	13,106	I c 3
8204	12,186	I b 6			

in the preceding tables; (3) a summary of the evidence in favor of each method, based upon (*a*) the appearance of the bands under high dispersion, and ordinary conditions of excitation (work of the author); and (*b*) the appearance of the bands under low dispersion, but unusual conditions of excitation (work of previous investigators).

SECTION I

The nitrogen lines of wave-length longer than λ 5100, comprising the First Deslandres' Group, fall into 57 similar groups of lines, called "bands." Each band contains several sets (usually five) of particularly heavy and close lines. These sets have been called the "heads" of the bands. That set in each band lying farthest to

the red usually ends abruptly on the red side, and has been called head I, the band being said to begin at this point, and to be degraded toward the violet.

In an ordinary band, such as is found in the Second Deslandres' Group of the nitrogen spectrum (λ 5060 to λ 2814) there are series of lines starting at the head, and proceeding with diminishing intensity toward the violet. Near the head, the lines of such a series are so related that successive frequency intervals form an arithmetical series. This is Deslandres' Law for band series. In the First Delandres' Group, however, there appear to be no relationships between the 250 or more lines forming each "band." This is *not*, therefore, an ordinary band spectrum.

Relationships first appear when we group together corresponding lines in successive bands, choosing one line from each band. We might take one line from the first (I) head of the λ 6623 band, another from the first head of the λ 6545 band, etc., and thus form a series. Under low dispersion the set of lines forming a head appears as a single broad line. Thus successive first heads were found to form a series satisfying Deslandres' Law— similarly successive second (II) heads, etc. Such a series extends over 10 to 15 bands, and then the interval between successive terms changes abruptly. Accordingly the ten or more bands represented in such a series have been classified as a "group of bands." The entire First Deslandres' Group is composed of five, and possibly six, such subgroups, which we have designated a to f respectively. Von der Helm decided that this was the best method for grouping the band heads, and arranged his data in this way. I shall therefore refer to it as the von der Helm arrangement, although it is not original with him.

The second arrangement of the bands was first suggested by Cuthbertson. In this the head of a band in one of the above groups is related, not to the adjacent *band*, but to a band in the adjacent band *group*. In the series thus formed we have only as many terms as we have band groups, and the spacing between terms is much greater than in the von der Helm arrangement. Since a series contains the head of only one band of a group, there are at least as many series as there are bands in a group. It is possible to form

11 such series having at least three terms each, and 6 more having only two terms each.

The reason for such a grouping is that the 17 series thus formed are identical in spacing with one another, and also with the five series into which the bands of the Second Deslandres' Group have been divided. Each series appeared to fulfil Deslandres' Law, and is known as Deslandres' First Progression. Each series, moreover, is displaced relative to the preceding one by a regularly increasing amount. Thus the corresponding terms of the several series form of themselves another set of series, also approximately obeying Deslandres' Law, and known as Deslandres' Second Progression.

Thus the entire set of the first heads of the bands in the First Deslandres' Group can be represented as a function of two parameters, p and n. The variation of n gives the First Progression, that of p the Second. Deslandres considered that both progressions obeyed his law, and wrote the complete formula

$$v = A + B(n + c_1)^2 + C(p + c_2)^2. \tag{1}$$

In this formula we can make a linear transformation of variables

$$k = \tfrac{1}{2}(p + n) \qquad l = \tfrac{1}{2}(p - n)$$

and obtain $f(k, l)$, also of second degree in each parameter, and so giving the ordinary Deslandres' Law when one parameter alone is varied. In such a $f(k, l)$ successive integral values of k (l remaining constant) give the heads of the successive bands of one group of the von der Helm arrangement. On the other hand, l has different values for successive band groups.

Fig. 1 may make this clearer. This figure gives the general position of the first head of every band, plotted with frequency as one co-ordinate and the value of p as the other. Any horizontal succession of heads, for which $p =$ constant, gives Deslandres' First Progression. The value of n for each head is plotted beside it, and any succession of heads for which $n =$ constant gives the Second Progression. The series $l =$ constant indicates one of the band groups of the von der Helm arrangement.
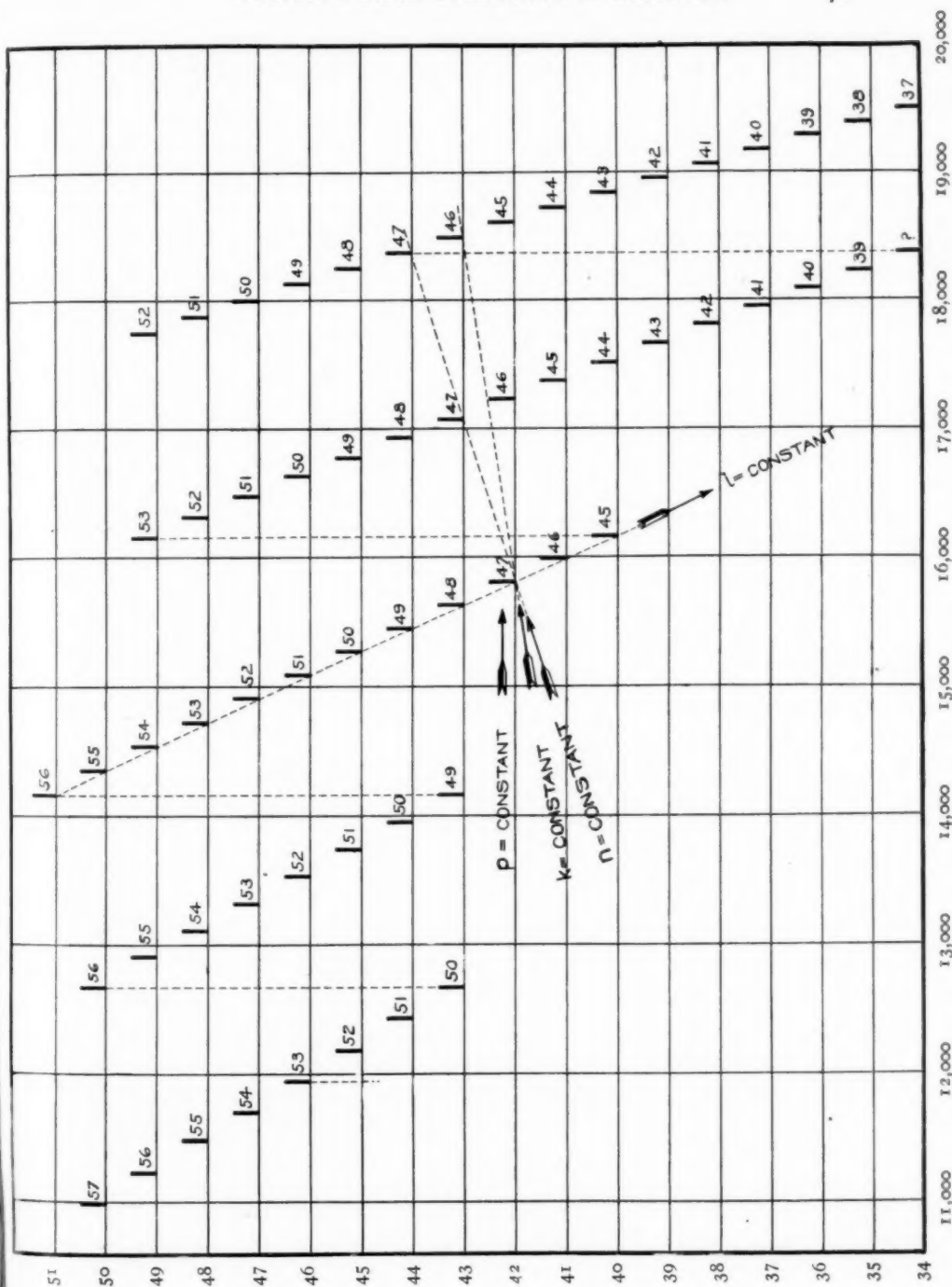
FIG. 1

SECTION II

That portion of the nitrogen spectrum under investigation appears to be formed of two superimposed spectra. One of these consists of lines of regular arrangement, the other of lines arranged irregularly. A graph of the lines of several bands of the e group indicates that perhaps 50 out of the 250 lines of each band belong to the regular spectrum. These sets of 50 lines have a similar appearance in each band. It is thus possible to identify corresponding lines in successive bands and to form them into series extending through one band group, and obeying Deslandres' Law as a first approximation. I shall call each of the 50 series thus formed a "simple" series.

The first heads of successive bands are composed mainly of several such series, and the general position of the first heads of successive bands, under low dispersion, forms roughly such a series. Table IV gives the simple series of longest wave-length in each band group. It is therefore composed of the "first" heavy line in each band, in the case of all the bands photographed under high dispersion. For the others the approximate position of the edge of the first head is used, as given in Tables II and III. Deslandres' Law demands that the first frequency differences, given in the fifth column, shall form an arithmetical progression, the second differences (sixth column) being a constant. The probable experimental error, in terms of frequency, varies from 0.04 at λ 5000 to 0.02 at λ 6800. Such an average error in the measurements, however, may cause an average variation four times as large in the second differences given in the last column.

Each series evidently obeys Deslandres' Law for the major portion of its extent, but deviates from this law near the violet end of the group. This is true for series in all band spectra, Deslandres' Law holding only near the head of a series. The only formula holding for an entire series is that of Thiele.[1] It contains eight undetermined coefficients and so is very difficult to work with. I have preferred to use simply Deslandres' Law, or a slight modification of it, and to note whether there was a regular deviation from this law.

[1] *Astrophysical Journal*, **6**, 65, 1897.

TABLE IV

DESIGNATION		λ (Air)	FREQUENCY (vacuo)	FIRST DIFFERENCE	SECOND DIFFERENCE
	p n				
f 16	34–37	5126.81	19,499.86		
				107.88	
f 15	35–38	5155.32	19,391.98		0.27
				108.15	
f 14	36–39	5184.24	19,283.83		1.22
				109.37	
f 13	37–40	5213.81	19,174.46		1.27
				110.64	
f 12	38–41	5244.07	19,063.82		1.39
				112.03	
f 11	39–42	5275.07	18,951.79		1.49
				113.52	
f 10	40–43	5306.86	18,838.27		1.32
				114.84	
f 9	41–44	5339.41	18,723.43		1.58
				116.42	
f 8	42–45	5372.82	18,607.01		1.65
				118.07	
f 7	43–46	5407.13	18,488.94		1.50
				119.57	
f 6	44–47	5442.32	18,369.37		1.62
				121.19	
f 5	45–48	5478.47	18,248.18		1.63
				122.82	
f 4	46–49	5515.59	18,125.36		1.64
				124.46	
f 3	47–50	5553.73	18,000.90		1.55
				126.01	
f 2	48–51	5592.88	17,874.89		0.52
				126.53	
f 1	49–52	5632.75	17,748.36		
e 15	35–39	5484.34	18,228.65		
				141.19	
e 14	36–40	5527.15	18,087.46		0.46
				141.65	
e 13	37–41	5570.78	17,945.81		0.69
				142.34	
e 12	38–42	5615.32	17,803.47		0.83
				143.17	
e 11	39–43	5660.84	17,660.30		1.07
				144.24	
e 10	40–44	5707.46	17,516.06		1.02
				145.26	
e 9	41–45	5755.19	17,370.80		1.23
				146.49	
e 8	42–46	5804.13	17,224.31		1.40
				147.89	
e 7	43–47	5854.40	17,076.42		1.32
				149.21	
e 6	44–48	5906.01	16,927.21		1.46
				150.67	

TABLE IV—*Continued*

DESIGNATION		λ (AIR)	FREQUENCY (vacuo)	FIRST DIFFERENCE	SECOND DIFFERENCE
	p *n*				
e 5	45–49	5959.05	16,776.54		1.43
e 4	46–50	6013.57	16,624.44	152.10	1.52
e 3	47–51	6069.66	16,470.82	153.62	1.51
e 2	48–52	6127.37	16,315.69	155.13	1.41
e 1	49–53	6186.73	16,159.15	156.54	
d 12	40–45	6185.22	16,163.09		
d 11	41–46	6253.00	15,987.91	175.18	1.35
d 10	42–47	6322.82	15,811.38	176.53	1.03
d 9	43–48	6394.63	15,633.82	177.56	1.21
d 8	44–49	6468.60	15,455.05	178.77	1.36
d 7	45–50	6544.88	15,274.92	180.13	1.34
d 6	46–51	6623.57	15,093.45	181.47	1.28
d 5	47–52	6704.75	14,910.70	182.75	1.43
d 4	48–53	6788.61	14,726.52	184.18	1.8
d 3	49–54	6875.5	14,540.5	186.0	
d 2	50–55	6968.0	14,347.6	192.9	
d 1	51–56	7059.6	14,161.2	186.4	
c 8	43–49	7059.6	14,161.2		
c 7	44–50	7165.0	13,953.0	208.2	
c 6	45–51	7274.0	13,743.8	209.2	
c 5	46–52	7386.1	13,535.3	208.5	
c 4	47–53	7505.6	13,319.9	215.4	
c 3	48–54	7624.8	13,111.5	208.4	
c 2	49–55	7742. (?)	12,913. (?)	198.5 (?)	
c 1	50–56	7887.	12,676	237. (?)	
b 8	43–50	7887	12,676	246	

TABLE IV—*Continued*

DESIGNATION			λ (AIR)	FREQUENCY (*vacuo*)	FIRST DIFFERENCE	SECOND DIFFERENCE
		p n				
b	7	44–51	8043	12,430		
					244	
b	6	45–52	8204	12,186		
					240	
b	5	46–53	8369	11,946		
					241	
b	4	47–54	8541	11,705		
					223	
b	3	48–55	8707	11,482		
					253	
b	2	49–56	8903	11,229		
					244	
b	1	50–57	9101	10,985		

In groups *d* and *e* it is occasionally doubtful what line forms the beginning of a new band.   In the first heads of the *f* group, however, there appears an extremely heavy doublet, the successive pairs of lines having not only the same relative intensity, but also a constant frequency difference.   The doublets thus form two simple series, of which that of longer wave-length has been used for the *f* group of Table IV.   In *f* 1 only one member of the doublet is present—that of shorter wave-length.   Hence it does not fit well with the other lines in Table IV.   I give in Table V the simple series formed from the more refrangible member of the doublet.

The first nine terms of this series can be fitted into the ordinary Deslandres' formula

$$v = A + B(m+c)^2 \qquad\qquad (2)$$

with an average difference between observed and computed values of 0.005 A.   For the *less* refrangible member of the doublet the corresponding average difference is 0.006 A, and the constants for this latter series are:

$$A = 22,900.627$$
$$B = -\quad 0.8000$$
$$c = +\quad 0.260$$
$$m = 80 \text{ to } 72$$

The beginning of the deviation from Deslandres' Law occurs, in both series, at a point of minimum intensity at *f* 10 (λ 5306).

TABLE V

DESIGNATION			λ (AIR)	FREQUENCY (vacuo)	FIRST DIFFERENCE	SECOND DIFFERENCE
		p n				
f	1	49–52	5632.754	17,748.361		
					127.701	
f	2	48–51	5592.514	17,876.062		1.669
					126.032	
f	3	47–50	5553.362	18,002.094		1.597
					124.435	
f	4	46–49	5515.239	18,126.529		1.629
					122.806	
f	5	45–48	5478.124	18,249.335		1.607
					121.199	
f	6	44–47	5441.981	18,370.534		1.615
					119.584	
f	7	43–46	5406.785	18,490.118		1.571
					118.013	
f	8	42–45	5372.496	18,608.131		1.584
					116.429	
f	9	41–44	5339.089	18,724.560		1.544
					114.885	
f	10	40–43	5306.529	18,839.445		1.479
					113.406	
f	11	39–42	5274.777	18,952.851		1.379
					112.027	
f	12	38–41	5243.782	19,064.878		1.448
					110.579	
f	13	37–40	5213.540	19,175.457		1.197
					109.382	
f	14	36–39	5183.970	19,284.839		1.364
					108.018	
f	15	35–38	5155.095	19,392.857		

At this same point the frequency difference of the doublet also begins to diminish. For these two reasons it appears that the f group consists really of two groups, having a point of coincidence at λ 5306. Table VI gives the frequency difference of the doublets for the entire f group.

I have thus far been unable to find any other strong series lying within the heads of the f group. In the d and e groups, however, there are at least 15 series, distributed among the three heads. In most of these the second difference remains approximately constant for six or eight terms; in a few it forms more nearly an arithmetical progression, the third difference being constant. Such a relation can be satisfied by adding one more term to Deslandres' Law, so that it reads:

$$v = A + B(m+c)^2 + C(m+c)^3 . \tag{3}$$

TABLE VI

	DESIGNATION						
	$f\,2$	$f\,3$	$f\,4$	$f\,5$	$f\,6$	$f\,7$	$f\,8$
Difference $\left(\text{in } \dfrac{10^8}{A}\right)$	1.173	1.193	1.167	1.157	1.161	1.176	1.123

	DESIGNATION						
	$f\,9$	$f\,10$	$f\,11$	$f\,12$	$f\,13$	$f\,14$	$f\,15$
Difference............	1.127	1.175	1.064	1.059	1.034	1.006	0.876

In the fifteen series the average difference of experimental and calculated values is slightly more than 0.01 A. In some cases it is over 0.02 A and evidently exceeds the experimental error of measurement. The lines forming the doublets in f are very difficult to measure correctly, because of their great intensity, and the nearness of adjacent lines. Yet they fit into series better than any other set of lines. Hence the deviations from formulae (3) or (2), in the case of other series are real, and not due to experimental errors.

The spacing arrangement in different series varies slightly, so that series often tend to cross one another, and this gives successive heads an entirely different appearance. This can best be shown by the five series in heads IV d. These five series include nearly two-thirds of all the lines present in these heads, and, with two exceptions, every strong line. Series δ and ϵ start from the same line and gradually diverge. Series δ, at the fifth term, breaks into a doublet, the components of which in turn diverge. The middle of the doublet is used for the last two terms. Such a sudden splitting of a line into a doublet is common in the series found in band spectra, and there are numerous examples of it in the spectrum under investigation. The five series are given in Table VII.

The foregoing portion of Section II has been concerned simply with the law followed by individual simple series, each being considered entirely independently. There are also relationships

TABLE VII

IV d α

λ (Air)	Frequency (*vacuo*)	First Difference	Second Difference
6758.054	14,793.111		
		184.265	
6674.908	14,977.376		1.460
		182.805	
6594.418	15,160.181		1.312
		181.493	
6516.403	15,341.674		1.340
		180.153	
6440.768	15,521.827		1.349
		178.804	
6367.416	15,700.631		1.251
		177.553	
6296.212	15,878.184		1.020
		176.533	
6226.978	16,054.717		1.165
		175.368	
6159.692	16,230.085		

IV d β

λ (Air)	Frequency (*vacuo*)	First Difference	Second Difference
6757.355	14,794.642		
		184.242	
6674.236	14,978.884		1.383
		182.859	
6593.739	15,161.743		1.411
		181.448	
6515.759	15,343.191		1.322
		180.126	
6440.150	15,523.317		1.311
		178.815	
6366.808	15,702.132		1.234
		177.581	
6295.606	15,879.713		1.127
		176.454	
6226.416	16,056.167		1.012
		175.442	
6159.114	16,231.609		

IV d γ

λ (Air)	Frequency (*vacuo*)	First Difference	Second Difference
6756.666	14,796.151		
		184.127	
6673.615	14,980.278		1.320
		182.807	
6593.155	15,163.085		1.340
		181.467	
6515.181	15,344.552		1.352
		180.115	
6439.590	15,524.667		1.272
		178.843	
6366.252	15,703.510		1.306
		177.537	
6295.077	15,881.047		

TABLE VII—*Continued*

IV *d* δ

λ (Air)	Frequency (*vacuo*)	First Difference	Second Difference
6755.948	14,797.724		
		184.040	
6672.954	14,981.764		1.368
		182.672	
6592.568	15,164.436		1.393
		181.279	
6514.687	15,345.715		1.435
		179.844	
6439.220	15,525.559		1.304
		178.540	
6366.010	15,704.099		

IV *d* ε

λ (Air)	Frequency (*vacuo*)	First Difference	Second Difference
6755.948	14,797.724		
		184.268	
6672.852	14,981.992		1.491
		182.777	
6592.423	15,164.769		1.293
		181.484	
6514.459	15,346.253		1.374
		180.110	
6438.887	15,526.363		1.273
		178.837	
6365.564	15,705.200		

between the spacing arrangement of simple series in different band groups. This can best be studied from the standpoint of the Cuthbertson arrangement.

In a two-parameter formula such as (1) there may be included one line from each band in the entire spectrum. It therefore comprises several simple series. The entire set of simple series, one for each band group, satisfying separately and collectively such a two-parameter formula I call a "complete" series. When the lines of any complete series are regrouped to form the p and n progressions, it appears that formula (1) is not the correct functional form. Table VIII shows this clearly. In this table I give only the average frequency intervals of the two progressions, using the data given in Table IV.

TABLE VIII

First Progression p = Constant			Second Progression n = Constant		
n	First Frequency Difference	Second Frequency Difference	p	First Frequency Difference	Second Frequency Difference
54			49		
	1615.0			1432.3	
53		25.6	48		27.3
	1589.4			1405.0	
52		29.9	47		28.0
	1559.5			1377.0	
51		28.9	46		27.8
	1530.6			1349.2	
50		29.0	45		27.8
	1501.6			1321.4	
49		29.6	44		28.0
	1472.0			1293.4	
48		29.6	43		28.6
	1442.4			1264.8	
47		29.7	42		28.5
	1412.7			1236.3	
46		29.9	41		28.9
	1382.8			1207.4	
45		30.2	40		29.4
	1352.6			1178.0	
44		30.3	39		29.7
	1322.3			1148.3	
43		30.8	38		30.3
	1291.5			1118.0	
42		31.2	37		31.0
	1260.3			1087.0	
41		31.6	36		31.0
	1228.7			1056.0	
40		32.4	35		33.4
	1196.3			1022.6	
39		31.3	34		
	1165.0				
38		34.5			
	1130.5				
37					

The second difference is an approximate arithmetical progression and requires a function of the type given in formula (3). Instead of formula (1) we must therefore use:

$$v = A + B(n+c_1)^2 + r(n+c_1)^3 + C(p+c_2)^2 + s(p+c_2)^3 . \tag{4}$$

Since the variation of both n and p has the same functional form, it follows that the variation of both together, such as we find in a simple series, has also this same form. For that reason it is possible to combine two simple series in order to determine the constants of a complete series. The two conditions imposed upon such a pair of

simple series are: (1) each simple series must fit formula (3); (2) both simple series must have the same third difference.

In formula (3) this third difference equals $6C$; in (4) it is $6(r+s)$. It is therefore the same for both simple series. When the constants of a complete series are thus determined, all other simple series included in the complete series have definite predicted positions.

If we now choose the simple series given in Table IV, using only the band groups for which we have accurate measurements (groups f, e, and part of d), it appears that all three simple series satisfy condition (1), but no two of them satisfy condition (2). It is therefore impossible to group them together into a complete series satisfying formula (4), and so the first lines of the first heads of all bands do *not* satisfy the Cuthbertson arrangement. Another way of stating this is that the several First Progressions are not identical with one another. This was evident in compiling Table VIII. There are eight intervals in this table whose values can each be derived from two different First Progressions (and similarly for the Second Progressions), using only accurate data. For these eight intervals the average difference of the two values is 0.2 A, more than ten times the experimental error.

In the I heads of the d group there are three heavy lines in all. The two of shorter wave-length form a doublet of the same constant frequency difference as that in the f group. This suggested the combination of these two series of doublets into two complete series, which should differ from one another only by a constant value. It appears that the two simple series formed from the doublets in the d group are compatible with those of the f group, and so this rearrangement into complete series is possible.

The simple series in the d group, of shorter wave-length, is given in Table IX.

Using the two simple series given in Tables V and IX, we get the following constants for the complete series. The derivation is rather laborious, and the computations were not made by a strictly least-squares method:

$$A = 22{,}108.476 \qquad r = +0.0245$$
$$B = -\ 18.0562 \qquad s = -\ .0254$$
$$C = +\ 17.2474 \qquad c_1 = +\ .3365$$
$$c_2 = +\ .7222$$

For p and n the derived values are respectively three and four units lower than Deslandres' values, which I have consistently used in designating the bands. This shows not only that the values of c_1 and c_2 (which define the "phase" of a series) are meaningless without more accurate data, but also that no deductions can be drawn from the exact value of $p-n$ for any band group.

TABLE IX

DESIGNATION		λ (AIR)	FREQUENCY (vacuo)	FIRST DIFFERENCE	SECOND DIFFERENCE
	p n				
d 4	48–53	6787.712	14,728.479		
				184.174	
d 5	47–52	6703.879	14,912.653		1.291
				182.883	
d 6	46–51	6622.658	15,095.536		1.306
				181.577	
d 7	45–50	6543.942	15,277.113		1.335
				180.242	
d 8	44–49	6467.634	15,457.355		1.352
				178.890	
d 9	43–48	6393.636	15,636.245		1.210
				177.680	
d 10	42–47	6321.797	15,813.925		

For eight terms from the f group, and seven from the d group, the average difference (obs.—calc.) is 0.005 A. For the less refrangible member of the doublet we have

$$A = 22,107.315$$

The other constants remain the same. For 14 terms the average difference (obs.—calc.) is 0.01 A.

By means of the constants given above we can obtain the theoretical position of corresponding simple series in all other band groups. From the position of the component simple series, in the heads of the d and f groups, we should expect the predicted series in the b and c groups to lie just to the violet of the rough measurements of the first heads in those groups. This is found to be the case, within the limits of experimental error. In the e group, however, where we have accurate data, there is no series in the predicted position. All series in I e have a slightly different spacing arrangement, and one of them gradually crosses the predicted series.

Thus the only Cuthbertson arrangement I have been able to get is between alternate rather than adjacent groups. As already pointed out, k $\{=\frac{1}{2}(p+n)\}$ and l $\{=\frac{1}{2}(p-n)\}$ are the parameters in the von der Helm arrangement, corresponding to p and n in the Cuthbertson arrangement. In this latter arrangement the first heads of all the bands are represented by integral values of p and n. In the von der Helm arrangement integral values of k give a simple series. If, however, we keep k constant, and give l successive integral values, we get corresponding first heads only in every alternate band group. (See series $k=$ constant on Fig. 1.) For the intermediate groups l has the value of an integer plus one-half, and cannot be satisfied by integral values of p and n. Therefore we might expect to find related simple series only in every alternate group. I have at present no other numerical evidence either for or against this view.

The previous discussion shows that many more lines can be fitted into series on the von der Helm arrangement than on the Cuthbertson. This naturally follows from the fact that each simple series involves only one parameter, while the Cuthbertson arrangement involves two. The individual series in different band groups should have related spacing arrangements, given implicitly by formula (4). The data show, however, that the relation is in general not accurate within the limits of experimental error.

One further point of interest is the continuity of successive band groups. The heads of the last band of one group practically coincide with those of the first band of the succeeding group. In this connection the band at λ 6186 is the most interesting in the entire spectrum. In this band we have at 6186.7 a head which agrees in its general position and appearance with the designation I e 1; similarly at 6185.2 a head I d 12. The entire appearance of the band is that of a d band, and it is doubtful whether the e group is represented save by I e 1, although I have recorded in Table I the lines in the vicinity of the theoretical position of II e 1 and IV e 1.

In the case of the e and f groups, the theoretical position of I e 16 is 5443.3, almost coinciding with the strong I f 6 head at 5442.3. Deslandres records I e 16 but there seem to be no lines

at this point resembling an e head. Again, however, the rough theoretical positions of these two heads almost coincide. For the other groups the coincidences are at 7059.6 and 7887. The data are so inaccurate here that the positions will fit equally well in either of the adjacent band groups. Considering that we have at least approximate coincidences at the four points mentioned above, several interesting relations follow.

The values of p and n at these points are:

	p	n		p	n		p	n		p	n
1	43–50		2	43–49		3	40–45		4	34–38	
	50–56			51–56			49–53			44–47	

Since the coincidence is between two heads of different band groups, the two values of $p-n$ at each point differ by unity. Two other unexpected facts, however, are: (1) that the discontinuity in p increases by unity at each succeeding point of coincidence; and (2) that the number of bands between points of coincidence increases by four, from group to group. This is also shown in Fig. 1. The coincident points are indicated by vertical dotted lines. The length of these lines gives the discontinuity in p. The number of bands between them is seen to increase by four, as one goes from red to violet. In group c there are six, in d ten, and in e fourteen bands.

If this rule were followed farther to the red we should expect only two bands in b, between the coincident bands 46–53 and 43–50. This would be the last group, the next one, by rule, having zero length. On the violet side we should expect nine more bands (including the coincident ones) in the f group, 25–28 coinciding with 36–38 of an unknown g group, and so on. In the next (h) group the last of the 26 predicted bands would have $p=-1$, $n=0$. Since the correct value of $p-n$ for any band group is indeterminate to at least one integer, it seems natural to suppose that all values of p should be raised by one integer.

We should then have a complete plan for the First Deslandres' Group. It would start, theoretically, at $p=0$, $n=0$, and would consist of seven groups of bands. The first head of some band near the end of each group would coincide approximately with the first head of a band in the next group. The number of bands

between coincidences would diminish by four, in each succeeding group. Some groups run past the points of coincidence and so overlap on each other.

Although groups *g* and *h* do not appear in the ordinary spectrum, Goldstein[1] believes he has seen the First Deslandres' Group, under certain low-temperature conditions, extending into the blue. Other investigators have been unable to verify this. Group *h* should start at λ 4430 and extend to λ 4530. Group *g* should extend from this latter point to λ 4890, and *f* from λ 4890 to λ 5442.8.

SECTION III

Under high dispersion successive bands have a very similar appearance, and this not only suggested to the author the formation of simple series, but also indicates the validity of the von der Helm arrangement of the bands. The general intensity of successive bands also varies continuously through a group of bands. All simple series were formed from lines of the same general appearance, and of a continuously varying intensity. The large number of possible series with approximately the same spacing is also good evidence of a connection between successive lines.

A few bands in group *e* have been measured and plotted beneath one another. It is these bands (λ 5900–λ 5700) that indicate the existence of some 50 simple series of lines, superimposed upon a much larger number of unrelated lines. Below λ 5700 all of the *e* series die out, save only those in the first heads. In the *d* group, however, the series extend to the last regular *d* band at λ 6185, and perhaps farther. In the case of the *f* group there seem to be no conspicuous series save the two mentioned in the first heads. This portion is the most irregular of the entire spectrum.

The only exceptions to the general rise and fall of intensity in the bands of one group are two very strong heads at λ 7072.8 and λ 6968.0. The latter lies at the predicted position of I *d* 2. The former lies somewhat to the red of I *c* 8. There is no apparent reason why either one should be strong.

On the other hand, all changes in appearance of the bands under changing physical conditions point to the Cuthbertson

[1] Goldstein, *Phys. Zeitschr.*, **6**, 14, 1905.

arrangement as the one indicating the actual physical connection between the sources of the radiation. Fowler[1] has shown that the spectrum of the active modification of nitrogen shows certain of the bands of the First Deslandres' Group greatly intensified, while the others are very faint or entirely lacking. The three strongest bands are those at λ 6253, λ 5804, and λ 5407 ($n=46$; $p=41$, 42, and 43), while the weaker bands on each side are those at λ 6323, λ 5854, λ 5442 ($n=47$; $p=42$, 43, 44) and at λ 6185, λ 5755, and λ 5373 ($n=45$, $p=40$, 41, and 42). Fowler has pointed out this evidence in favor of the Cuthbertson arrangement.

The fact that apparently the entire band is increased in intensity may point to a further relation, not included in the Cuthbertson. It would be very interesting to photograph this spectrum under high dispersion and to note whether *all* the lines of a band were intensified, or only those belonging in series.

Angerer[2] has made an exhaustive study of the First Deslandres' Group at low temperature. I have made no critical study of his results, and cannot well do so until I have my own measurements completed. Several points, however, are worth noting.

At low temperature the heads of a band are far more intense, relative to the rest of the band, than at ordinary temperature. This is especially true of the III heads which, at high temperature, escape detection in many bands—not having been measured at all by von der Helm. But they are particularly strong at low temperature. This would point to an independence between the series lying within the heads of the bands, and other series.

At low temperature the entire spectrum is relatively much fainter than at room temperature. Aside from two small groups of lines in the green, the only exceptions to this statement are the first heads of the three bands λ 6623, λ 6070, and λ 5593 ($n=51$; $p=46$, 47, and 48). The second of these is even more intense at low temperature, while the other two are fully as intense. Here again we have evidence in favor of the Cuthbertson arrangement.

There is one additional fact pointing to a general relationship between the heads of all the bands. The frequency difference of

[1] Fowler, *Proc. Roy. Soc.*, **85** A, 377, 1911.

[2] *Ann. d. Phys.*, **32**, 549, 1910.

the rough position of the I and IV heads is a constant for all bands from λ 5100 to λ 9100, although the length of the bands more than doubles within this range. The maximum variation of the difference is 7 units (from $v=68$ to 61). The frequency difference of the I and II heads, except in the f group, is also practically constant. I cannot recall having previously seen this fact explicitly stated.

This relation of the heads is what we should expect if the bands were composed of a number of identical series of lines. It seems evident that all possible series have *very closely* the same spacing, but it is also certain that the spacing is *not* identical.

Sections II and III may be summarized in the statement that numerical relationships among the lines of the First Deslandres' Group favor the von der Helm method of grouping, while changes in the bands under varying physical conditions of the source all point to the Cuthbertson method as the significant one.

CONCLUSIONS

1. The First Deslandres' Group of the positive band spectrum of nitrogen consists really of two spectra, one composed of a large number of superimposed series of lines, the other quite irregular.

2. The similarity in the spacing of all series gives the banded appearance of the spectrum, the length of a band being the distance between two successive lines of each series.

3. The so-called "heads" of the bands are formed by groups of particularly heavy lines, accompanied by more or less continuous radiation.

4. It is possible to fit a greater number of lines into the simple series of the von der Helm arrangement of bands than into the more complex two-parameter formula indicated by the Cuthbertson arrangement. All physical changes in the spectrum, however, favor the latter arrangement.

5. Simple series of lines, running through one band group of the von der Helm arrangement, obey Deslandres' Law for at least the first few bands, but later show a large and systematic deviation from it.

6. The First and Second Progressions of the Cuthbertson arrangement fit approximately into a formula containing both

the second and third powers of the parameter, but will *not* fit the simpler second-power formula of Deslandres' Law.

7. The successive band groups have certain heads which approximately coincide, and these points of coincidence show regularities which enable the entire set of bands of the First Deslandres' Group to be arranged so as to indicate a definite plan for the group.

The experimental part of the investigation is the resolving, for the first time, of the 39 bands between λ 5000 and λ 6800 into about 6400 lines, and the measurement of a portion of these lines with an average error of 0.01 A or less.

In conclusion the author wishes to express his thanks to Professor C. E. Mendenhall for the many helpful suggestions offered during the progress of this investigation.

DEPARTMENT OF PHYSICS
UNIVERSITY OF WISCONSIN
August 1913

NOTE ON THE RELATIVE INTENSITY AT DIFFERENT WAVE-LENGTHS OF THE SPECTRA OF SOME STARS HAVING LARGE AND SMALL PROPER MOTIONS[1]

By WALTER S. ADAMS

One of the methods proposed by Professor Kapteyn for the investigation of the question of the absorption of light in space is the comparison of the intensity at several different wave-lengths of the spectra of stars which are known to be near the earth with such as are very distant. The effect of general absorption or scattering, so far as is known, always increases toward shorter wave-lengths, and if such absorption is present in space we should expect the spectrum of the more distant star to fall off more rapidly in intensity toward the violet end of the spectrum than would the spectrum of the nearer star.

It is clear that for the purpose of making such a comparison a marked advantage will be gained if the spectra of the two stars are obtained upon the same plate, since the two spectra are then treated precisely alike as regards development, and some of the difficulties connected with the possible effect of difference of development upon different portions of the spectrum are eliminated. Accordingly we have adopted this procedure in the case of a number of photographs obtained for this purpose with the Cassegrain spectrograph. In a few cases the spectra of the two stars to be compared are photographed side by side through the star window in the occulting bar, a comparison spectrum being added for convenience in the determination of wave-lengths. More recently, however, we have used no comparison spectrum and have photographed but one star through the central window. Two exposures of somewhat different lengths are made upon the second star, one through each portion of the comparison spectrum window. The spectra extend from

about λ 4000 to λ 5000, and in securing the photographs the attempt has been made to obtain approximate equality of density for the two spectra at the less refrangible end.

Two conditions are essential to make a comparison of this sort of value: (1) The spectral types of the two stars must be practically identical. (2) The photographs must be taken with the stars at very nearly the same zenith distance, and with no appreciable variation in the transparency of the sky.

The investigation at this observatory of the radial velocities of a large number of stars with measured parallaxes and large proper motions has provided ample material for the selection of suitable spectra among the nearer stars. For the more distant stars it is necessary to depend upon proper motions alone, and the spectra of a considerable number of stars of small proper motion distant about $90°$ from the apex of the sun's motion, originally obtained for radial velocity determinations, are available for comparison with the parallax stars. The two spectra are compared side by side under a Hartmann spectro-comparator, and are rejected unless the agreement is essentially complete, line for line. Anyone who has classified stellar spectra is familiar with the rapid change in intensity at the violet end of the spectrum with even a comparatively small change of type.

Although the total number of photographs so far obtained is small, the results are of sufficient interest to deserve a few words of comment. Out of 20 pairs of stars investigated, two pairs are of type B8, one A0, one F4, one F7, two G5, two G6, one G8, seven K0, one K2, one K4, and one K6. Of these the pairs of stars of types B8, A0, and F4 show no appreciable relative difference between the two ends of the spectrum, and the same is true of one pair of type G6 and one of type K0. The remaining fourteen pairs all show a marked difference, which in some cases is very great. In every case the star which is relatively faint at the violet end of the spectrum is the star of small proper motion. The spectra of five of these fourteen pairs is shown in Plate IV, enlarged about eight times from the original negatives. The sections *a* and *e* show the two star spectra side by side with a comparison spectrum above and below. The other three sections show one star in the center and

PLATE IV
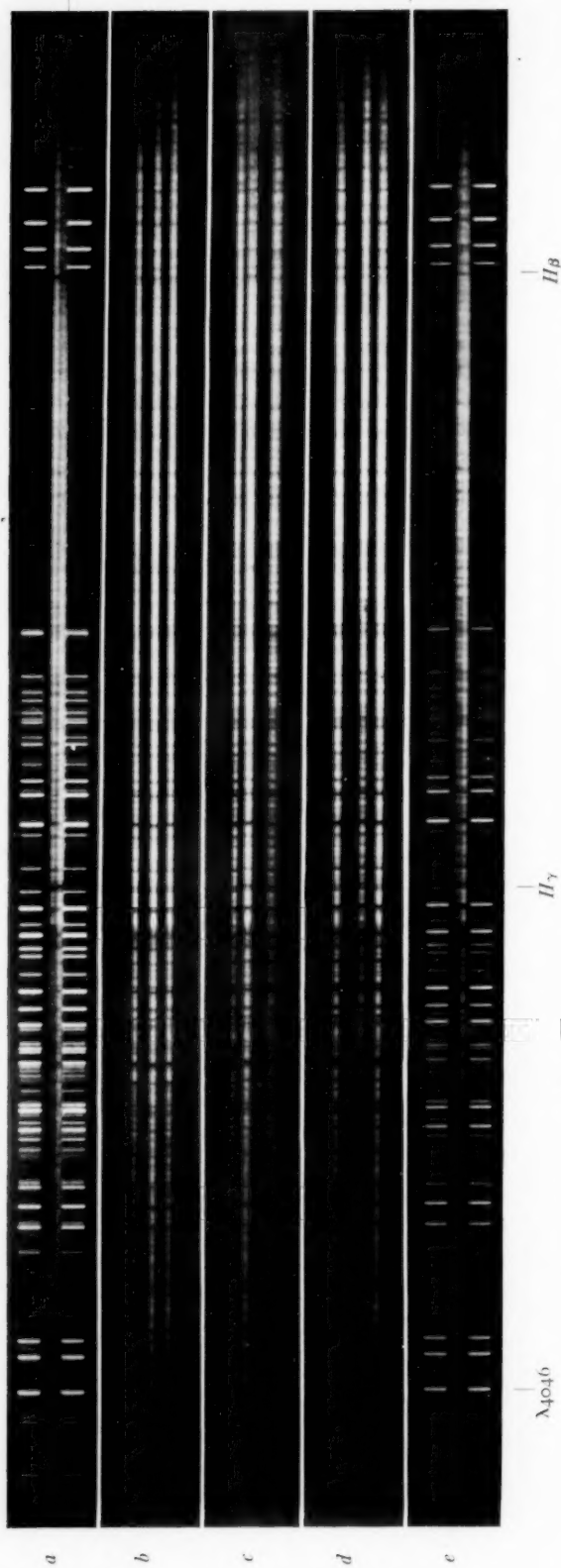
COMPARISON OF INTENSITIES IN THE VIOLET OF SPECTRA OF STARS OF LARGE AND SMALL PROPER MOTION

$H\beta$

$H\gamma$

$\lambda 4046$

two spectra of the second star, one on either side.   The data for
these stars are as follows:

		Star		Mag.	Spectrum	μ	π
a........	{ Above	*Boss*	3542	5.7	F7	0."009	
	{ Below	*Brad.*	1433	5.9	F7	0.19	+0."11
b........	{ Outside	*Boss*	434	6.0	K0	0.11	
	{ Inside	*54 Piscium*		6.1	K0	0.59	+0.15
c........	{ Outside	*Boss*	430	6.1	K0	0.005	
	{ Inside	*Brad.*	3212	6.2	K0	0.42	+0.15
d........	{ Outside	*Brad.*	3077	5.6	K4	2.10	+0.16
	{ Inside	*Boss*	6123	5.8	K4	0.020	
e........	{ Above	*Boss*	3922	5.8	G5	0.027	
	{ Below	*Boss*	4032	4.8	G5	0.364	

The use of proper motion as a measure of distance, however
valuable in the case of the average of a large number of stars, is of
course by no means conclusive in the case of an individual star, and
this consideration may well apply to some of the stars in the list.
For the six pairs of stars which show no marked effect in their spectra
only a single parallax determination is available, the relative dis-
tances of the remaining pairs being based upon proper motions
alone.   For three of, these, those of types B8 and Ao, the proper
motion of the star supposed to be the nearer amounts to only three
times that of the more distant star, and the results for these pairs
accordingly are of little weight.

The evidence of this small amount of material is much too slight
to warrant any extended discussion of its application to the problem
of the absorption of light in space.   The points of interest are:
first, that two stars having the same type of spectrum may differ
very greatly in the relative intensity of different portions of their
continuous spectra; second, that in no case is the more distant
star relatively stronger in the violet portion of the spectrum, but in
a considerable majority of cases is weaker.   It is clear, moreover,
that the results so far found may be explained equally well as an
effect of absolute brightness as of absorption of light in space.
Since the stars compared are of nearly the same apparent magnitude

the more distant star must be intrinsically much the brighter, and probably the more massive as well, since the spectra, and hence no doubt the physical conditions in the stellar atmospheres, are very similar. If the atmosphere of the more massive star absorbs more strongly than that of the smaller star, which seems fairly probable from physical considerations, this would account for the differences observed. In order to separate the effect of absolute magnitude from that of absorption of light in space, Professor Kapteyn has prepared an observing-list of pairs of stars of the same type of spectrum but of greatly different absolute magnitude, which are at the same distance from the earth, as in the case of physically connected double stars, or for which the parallaxes are approximately equal. These stars would be affected identically by absorption of light in space, and any observed differences in relative intensity in different parts of their spectra would necessarily be due to the effect of absolute brightness. The results obtained from a very few photographs of such pairs of stars as yet show nothing conclusive, although they point to no very definite effect due to this source. As an illustration, a photograph of the spectra of the stars a *Aurigae* and *Groombridge 884* shows little or no difference in the relative intensity of the violet and red ends of the spectrum. According to the measured parallaxes these stars differ by 7.6 magnitudes in absolute brightness, and with every allowance made for uncertainties in the values of the parallaxes the difference must still be very great.

It is evident even from the results given in this brief communication that the method of comparing the actual spectra of stars has several advantages over most other methods when the stars are sufficiently bright to enable its use. The fact that stars of identical spectra may be selected, and that the comparison may be made over a wide range of wave-length, is of especial value where the difference to be investigated is small.

MOUNT WILSON SOLAR OBSERVATORY
December 15, 1913

SECONDARY STANDARDS OF WAVE-LENGTH, INTER-NATIONAL SYSTEM, IN THE ARC SPECTRUM OF IRON ADOPTED BY THE SOLAR UNION, 1913

By H. KAYSER, J. S. AMES, H. BUISSON, F. PASCHEN

In continuation of the list of international standards of wave-length taken from the iron arc the lines contained in the accompanying list were adopted by the International Union for Co-operation in Solar Research, held in Bonn, August 1, 1913:

The following other propositions of the committee have been adopted by the Union: as some lines of the iron arc taken in atmospheric air are of poor quality, the committee recommends for the determination of tertiary standards the following conditions of the arc: (1) The length of the arc should be 6 mm. (2) For wave-lengths greater than 4000 A the current should be 6 amperes, and 4 amperes or less for the shorter wave-lengths. (3) A continuous current should be used with 220 volts, with iron rods 7 mm in diameter as electrodes. The arc should be vertical, the positive electrode being the upper one. (4) The middle part of the arc in its axis for a length of 2 mm should be used. (5) Only the lines belonging to the groups *a, b, c, d* of the Mount Wilson classification of the iron lines should be used, at least for that part of the spectrum for which this classification exists.

On Mount Wilson a slit perpendicular to the axis of the arc has given as good results as a slit parallel to the arc.

When the committee in its last report recommended that laboratories and observatories possessing concave gratings of first quality should be invited to measure tertiary standards from the iron arc, it was not the intention to exclude the use of plane gratings. On the contrary, the committee wishes to secure the collaboration of all those possessing plane or concave gratings or prisms with sufficient dispersion and resolution.

The committee recommends that many more secondary standards should be determined than are given in the published tables, in order to make the measurements by interpolation more easy and

more exact.   It would be a great advantage to have some hundred secondary standards instead of a few.

International Standards	Fabry and Buisson	Pfund	Burns	Eversheim
6750.163	....	....	.164	.162
6678.004	....	.004	.000	.008
6592.928	....	.928	.925	.931
6546.250	....	.251	.247	.252
5709.396	.396	.396	.395	....
4233.615	.615	.615	....	.616
4191.443	.441	.445	....	.444
4147.676	.677	.677	....	.674
4134.685	.685	....	....	.685
4118.552	.552	.551	....	.552
4076.642	.641	.644	....	.642
4021.872	.872	.871	....	.872
3977.746	.745	.745	....	.747
3935.818	.818	....	....	.818
3907.937	.938	....	....	.936
3906.482	.481	.483	....	....
3865.527	.526	....	....	.528
3850.820	.820	....	....	.820
3843.261	.261	.261	....	....
3805.346	.346	.346	....	.347
3753.615	.615	.616	....	.614
3724.380	.379	.380	....	.380
3677.629	.628	.630	....	.630
3676.313	.312	....	....	.314
3640.392	.391	.392	....	.392
3606.682	.681	....	....	.683
3556.881	.879	.883	....	.882
3513.821	.820	.821	....	.822
3485.345	.344	.346	....	.346
3445.154	.155	.154	....	.154
3399.337	.337	.338	....	.337
3370.789	.789	.788	....	.791

NICKEL-LINES TAKEN BETWEEN RODS OF NICKEL

International Standards	Fabry and Buisson	Pfund	Burns	Eversheim
5892.882	.882	.882	....	.881
5857.759	.760	.757	....	.759

STUDIES OF THE NOCTURNAL RADIATION TO SPACE

SECOND PAPER

By ANDERS ÅNGSTRÖM

I. RADIATION TO DIFFERENT PARTS OF THE SKY

In a previous paper[1] an account was given of some measurements showing the influence of water-vapor on atmospheric radiation. In the historical survey of that paper, I referred to the important investigations of Homén and mentioned his measurements of the nocturnal radiation to different parts of the sky. As Homén's measurements afterward have been employed in extending observations of the radiation toward a limited part of the sky to the whole sky and as the question itself seems to be of interest for the knowledge of the atmospheric radiation in its dependence on other conditions, it was found valuable to investigate in what degree this distribution of radiation over the sky is subjected to variations. For this purpose the arrangement shown schematically in Fig. 1 seems to be a natural one.

To the Ångström nocturnal compensation instrument can be attached a half-spherical screen, *abcdef*, whose radius is 7.1 cm. From this screen can be removed a spherical cap *cd*, which leaves a hole of 32° solid angle open to the sky. The screen is brightly polished on the outside, but on the inside blackened in order to avoid multiple reflections.

The instrument with this screen attached to it was pointed to different parts of the sky, and the zenith angle was read on a circular scale as is shown in Fig. 1. The value of the radiation within the solid angle *csd* (32°) was obtained in the usual way by determining the compensation current through the black strip.

This arrangement has two obvious advantages over a bolometer arranged in a similar way. In the first place the instrument is very steady and quite independent of air currents, because both strips here are exposed in exactly the same way. The sensitiveness of

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[1] *Astrophysical Journal*, **37**, 305, 1913.

the instrument is further quite independent of the position of the strips, it being possible to turn the instrument over at different angles, without change in the sensitiveness. Everyone who is familiar with bolometric work knows the difficulty that sometimes arises from the fact that the sensitiveness of the bolometer changes with its position, the conductivity of heat from the strips through the air being different for vertical and for horizontal positions.
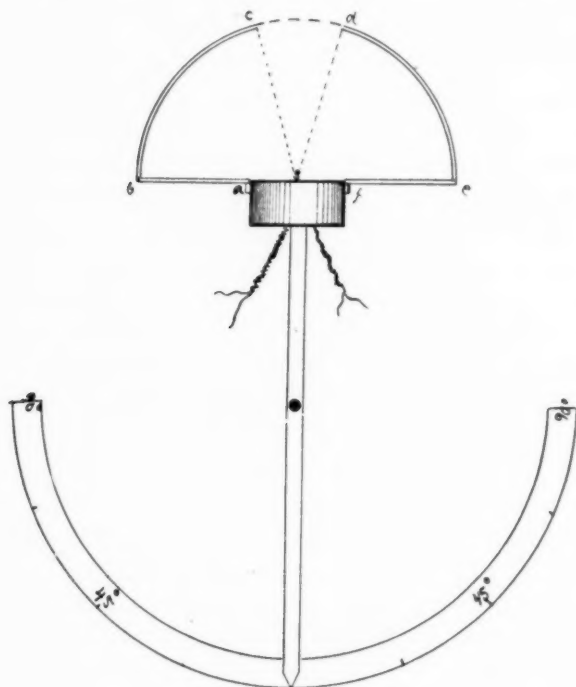

FIG. 1

In order to diminish the error that may arise from the fact that different parts of the strips radiate in somewhat different directions and under slightly different solid angles, the instrument was always turned over so that the strips were parallel to the earth's surface. In such a case, we may regard the above-mentioned influence of the dimensions of the strips as negligible, the strips being small in comparison with the radius of the hemispherical screen.

The results of these measurements for three different nights are contained in Table I. In order to obtain from these values a

TABLE I

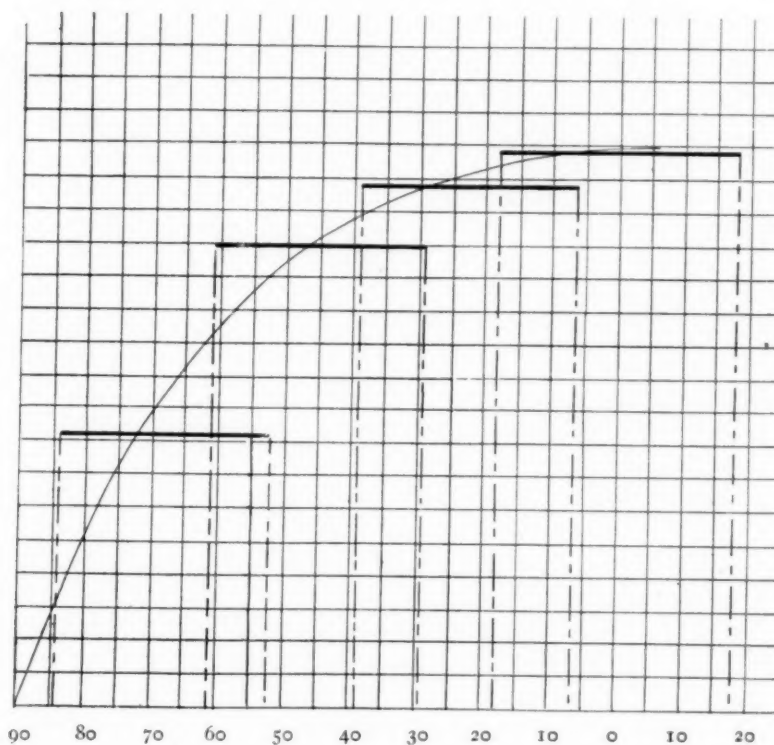| Date	Humidity	Total Rad.	o°	½ 45°	45°	¾ 45°	Temp.	Curve
1912								
23:8........	3.84	0.192	0.0173	0.0167	0.0164	0.0120	20.8	I
30:8........	7.10	0.157	0.0158	0.0147	0.0127	0.0062	20.3	III
4:9........	4.98	0.169	0.0168	0.0157	0.0140	0.0086	11.1	II
20:8........	13.24	0.145	0.0153	0.0151	0.0126	0.0041*	18.9	—

* Doubtful.

FIG. 2

more detailed idea about the effective radiation to different parts of the sky, I proceeded in the following way. In a system of co-ordinates, where the zenith angle is plotted along the *x*-axis,

the magnitude of the radiation along the _y_-axis, every measurement with the instrument corresponds evidently to an integral extending over $32°$ and limited by the _x_-axis and a certain curve, that is, the distribution curve of radiation. If the measurements are plotted as rectangular surfaces, whose widths are 32 and

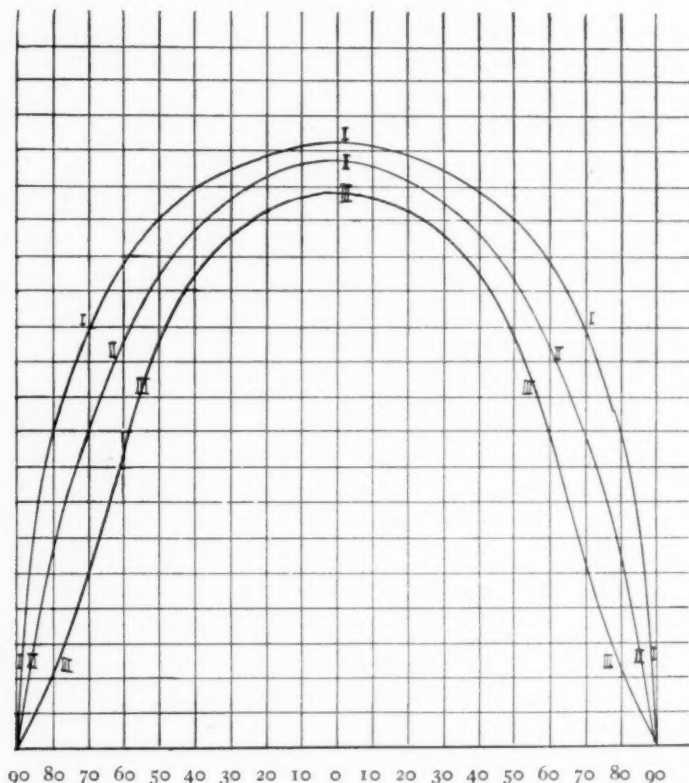90 80 70 60 50 40 30 20 10 0 10 20 30 40 50 60 70 80 90

FIG. 3

whose heights are proportional to the magnitude of the radiation, we shall obtain from the observations a system of rectangles like those in Fig. 2. A curve drawn so that the integrals between the limits corresponding to the sides of the rectangles are equal to the areas of these rectangles will evidently be a curve representing the radiation as a function of the zenith angle.

Fig. 3 shows these curves for three different occasions, corresponding to different values of the total radiation and to a different pressure of the atmospheric water-vapor.

If we consider these curves in connection with the corresponding water-vapor pressure, we shall arrive at the following conclusions:

1. *An increase in the water-vapor pressure will cause a decrease in the effective radiation to every point of the sky.*

2. *This decrease is much larger for large zenith angles than for small ones.*

I have shown in my first paper on this subject how the total effective radiation can be written as a function of the water-vapor pressure at the earth's surface. An increase in the water-vapor pressure produces a decrease in the effective nocturnal radiation according to a logarithmic law.

The observations described in the present paper show in what way this decrease is produced. The nearer the observation approaches the horizon, the more effectively is the radiation influenced by the density of the radiating atmosphere. When the zenith angle approaches a value of 90° the effective radiation approaches zero in a way that indicates that the atmosphere for large zenith angles radiates almost like a black body.

If we regard the atmosphere as a plane parallel layer, having uniform density, ρ, and a temperature uniformly equal to the temperature of the earth's surface, the radiation of a certain wave-length in different directions, may be expressed by

$$I_\lambda = C e^{-\gamma \cdot \frac{\rho}{\cos \phi}} \tag{1}$$

where C and γ are constants and ϕ is the zenith angle. For another density, ρ^1, of the radiating atmosphere we have:

$$I_\lambda' = C e^{-\gamma \frac{\rho^1}{\cos \phi}} \tag{2}$$

and from (1) and (2)

$$\frac{I_\lambda}{I_\lambda'} = e^{-\gamma \left[\frac{\rho - \rho^1}{\cos \phi}\right]} \tag{3}$$

If ρ is bigger than ρ^1, I_λ will always be less than I_λ'. It is easy to see from the relation (3) that the ratio between I_λ and I_λ' diminishes as the zenith angle approaches 90°. If we have to deal with

the total radiation of many different wave-lengths, the equations (1) and (2) must be written as sums of several terms differing from another in respect to the constants C and γ. The expression for the ratio $\dfrac{I_\lambda}{I_\lambda'}$ will be more complicated, and it is only under special conditions that the ratio diminishes with an increase in the zenith angle. The observations show that these conditions are here fulfilled.

The curves in Fig. 3 represent the effective radiation within the unit of solid angle in different directions from a surface perpendicular to the radiated beam. From these curves we can compute the radiation from a horizontal surface like the earth's surface, to the different zones of the sky. We have therefore to multiply

TABLE II

↓ Observer ↓	$0°.-22°30'$	$22°30-45°$	$45°-67°30$	$67°30-90°$
Homén...................	1.00	0.93	0.87	0.61
Ångström I............	1.00	0.94	0.86	0.60
" II..............	0.99	0.92	0.75	0.41
" III.............	0.97	0.91	0.65	0.23

every single value of sin ϕ · cos ϕ by a constant, whose value for the present does not interest us. In such a way the curves of Fig. 4 are obtained. Fig. 4 includes a dotted curve whose ordinates are everywhere proportional to a constant multiplied by sin ϕ · cos ϕ. From the data of Fig. 4, Table II is calculated, which also gives a comparison between the values here obtained and the values found by Homén. In Table II are given the ratios between the values governed by the observations and the values obtained from the simple sin-cos law, that is, for a case where a horizontal surface radiates directly to a non-absorbing space, the radiation assumed to be *one* for zenith angle 0°.

All the observations just described were made at Bassour, Algeria, at a height of 1160 m above sea-level. The sky was clear and appeared perfectly uniform.

II. THE DIFFUSE RADIATION FROM THE SKY DURING THE DAYTIME

In the daytime, the radiation exchange between the sky and the earth is complicated by the diffuse sky radiation of short wave-

length that comes in addition to the temperature radiation of the sky. If this diffuse radiation is stronger than the effective temperature radiation to the sky, a black body like our instrument will receive heat. In the opposite case it will lose heat by radiation.

If one attempts to measure this positive (from sky to earth) or negative radiation with the Ångström nocturnal compensation
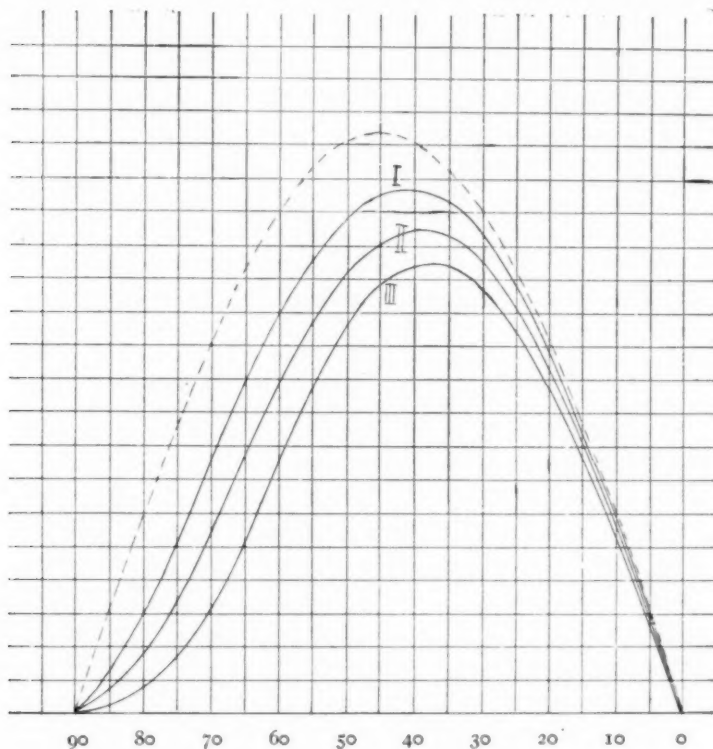

FIG. 4

instrument, the sun being carefully screened off, such an attempt meets with a systematic difficulty. The bright metal strip has namely a less reflecting power for the diffused radiation of short wave-length than for the longer heat waves and we can no longer apply the instrumental constant k, which only holds for long waves such as we have to deal with in the measurements of the nocturnal radiation. The reflecting power of the strips being about 92 per

cent for waves longer than 2 μ, and only about 60 per cent for waves of 0.5 μ length (a mean value of the wave-length of the diffused sky radiation), an application of the constant k to daylight measurements, will evidently give a value of the sky radiation that is about 30-35 per cent too low.

On several occasions during the summer of 1912, I had the opportunity to make sky-light measurements with the Ångström instrument and with an instrument founded on the same principle, but modified for the purpose of measuring diffuse sky radiation. This latter instrument is briefly described by Abbot and Fowle[1] in their interesting paper, "Volcanoes and Climate," where the effect of the diffusing power of the atmosphere on the climate is fully discussed. Both the strips are in this instrument blackened.

TABLE III

RADIATION OF THE SKY

	Sept. 5	Sept. 6	Sept 7	Mean
Before sunrise.............	−0.169	−0.205	−0.208	−0.194
Noon.....................	+0.062	+0.092	+0.047	+0.067
After sunset.............	−0.208	−0.225	−0.220	−0.218
Total sky radiation....	+0.250	+0.307	+0.261	+0.273

Instead of being side by side, the strips are here placed one above the other beneath a thin horizontal plate of brass. In the use of the instrument, a blackened screen was placed beneath it so that the lower strip was exchanging radiation only with this screen, which subtended a hemisphere. The upper strip was exchanging radiation with the whole sky. The radiation was computed from the current necessary to heat the upper strip to the same temperature as the lower one.

Even in the use of this instrument in its original form, it is difficult to avoid a systematic error. This error arises from the difficulty of protecting the screen, with which the lower strip exchanges radiation, from absorbing a small portion of the incoming radiation and in this way giving rise to a heating of the lower strip. I have reasons, however, to believe that the error arising from this

[1] *Smithsonian Miscellaneous Collections*, **60**, 29, 1913.

cause is not larger than 10 to 15 per cent. As well in this instrument as in the original Ångström nocturnal instrument, the error, when we attempt to measure the sky radiation during the day, is in the direction to make this radiation appear weaker than it really is.

Table III gives some results of the measurements with the last named instrument. These measurements were made by C. G. Abbot with the author's assistance and are published in the paper "Volcanoes and Climate," already referred to. With Mr. Abbot's permission they are published here also. My measurements of the nocturnal radiation during preceding and following nights are given in the same place. The total diffuse sky radiation is calculated on the assumption that the effective temperature radiation during the daytime is a mean of the morning and evening values determined by the nocturnal apparatus. The sky was perfectly uniform during the observations but was overdrawn by a weak yellow-tinted haze, by Abbot ascribed to the eruption of Mount Katmai in Alaska. The energy of the direct solar beam at noon was for all three days 1.24–1.25 cal. From Table IV may be seen that, under the conditions described, there was always an income of radiation from the sky, indicating that the diffuse radiation from the sky was always stronger than the outgoing effective temperature radiation.

The measurements with the Ångström nocturnal instrument on two different occasions showed in one case no appreciable radiation in any direction and in the other case a weak positive radiation from the sky. If in order to correct for the reflecting power of the strips for short waves we multiply the corresponding nocturnal temperature radiation (0.17–0.20) by 1.5, we shall obtain a result that is in pretty close agreement with the determinations with the modified instrument.

The result is different from that obtained by Homén. Homén draws from his observations at Lojosee in Finland the conclusion that there is an effective radiation from earth to sky even in the daytime. Our result is consistent, however, with Lo Surdo's measurements at Neapel in 1909, where he found a positive incoming radiation from the sky in the daytime.

The surplus of the scattered sky radiation over the effective temperature radiation to the sky, is, however, not so great but that one may very well imagine cases where the direction of heat transfer may be reversed. Under conditions where the temperature radiation to the sky is extraordinarily large, that is, when the air is unusually dry, and at the same time the scattering power of the atmosphere is low, it may very well be possible that the outgoing temperature radiation becomes the larger.

The measurements described were made under conditions where the temperature radiation must have been more than usually strong, even if the scattering power of the atmosphere at the same time was somewhat greater than one generally finds at this place and in this climate (Algeria). All taken into account, I am inclined to think that the heat transfer in the middle of the day generally goes in the direction from sky to earth and that the observations of Homén were made under conditions that must be regarded as exceptional.

NOTE ON INSTRUMENTS AND METHOD OF OBSERVATION

The observations of the nocturnal radiation, *described in this and a previous paper*, were carried out with two Ångström compensation instruments No. 17 and No. 18, made by G. Rose, Upsala. The constants of these instruments, determined at the Physical Institute of the University of Upsala, were respectively 10.4 and 11.1. The readings of the instruments were compared on numerous occasions. The values of the radiation obtained with the two instruments never differed by more than $2\frac{1}{2}$ per cent. All the values given in the tables are generally means of three independent readings. *Special attention was always given to see that the instrument was at constant temperature, before the strips were exposed to the sky.* If the instrument is removed in the free air, it requires between 10 and 15 minutes before the state of equilibrium is reached.

The current used for compensation of the heat lost by radiation (0.10–0.15 amp.) was measured by a Siemens and Halske milliammeter.

The water-vapor pressure was computed from readings of wet and dry thermometers, graduated in Fahrenheit degrees and made by H. T. Green, Brooklyn (Slingpsychrometer 150*a*).

CORNELL UNIVERSITY
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